MR2231011 (2007f:82031) 82B40 (47G20 76P05 82C40)
Baranger, Céline (F-ENSET-AM); **Mouhot, Clément** (F-ENSLY-PM)
Explicit spectral gap estimates for the linearized Boltzmann and Landau operators with hard potentials. (English summary)
Rev. Mat. Iberoamericana **21** (2005), *no. 3,* 819–841.

The authors study the spectral properties (spectral gap) of the linearized Boltzmann and Landau collision operators with hard potentials from a new perspective based on geometrical properties of the whole collision operators. The spectral theory for the linearized Boltzmann and Landau operators has been studied extensively by many other people using perturbative methods, symmetry arguments, Fourier methods, or cancellation methods. This geometric approach covers all the previous results for spectral gaps, for hard potentials, with or without angular cutoff, and moreover provides explicit and concrete estimates on spectral gaps.

The main idea of the proof is to reduce the case of hard potentials to the Maxwellian case, for which explicit estimates are already known. In order to do so, the authors prove the entropy-entropy dissipation type of inequality, first proposed by Villani, in the context of the Landau equation. The difficulty is to deal with the cancellations of the kinetic kernel on the diagonal $v = v_*$, and to get around it the authors introduce well-chosen intermediate collisions based on the geometry of pre- and post-collisions.

Reviewed by *Hyung Ju Hwang*

References

1. Bobyl"ev, A. V.: The theory of the nonlinear spatially uniform Boltzmann equation for Maxwell molecules, In Mathematical physics reviews, Vol. 7, 111–233. Soviet Sci. Rev. Sect. C Math. Phys. Rev. **7**, Harwood Academic Publ., Chur, 1988. MR1128328 (92m:82112)
2. Caflisch, R. E.: The Boltzmann equation with a soft potential. II. Non- linear, spatially-periodic. Comm. Math. Phys. **74** (1980), no. 2, 97–109. MR0576265 (82j:82040b)
3. Carrilo, J., McCann, R. and Villani, C.: Kinetic equilibration rates for granular media and related equations: entropy dissipation and mass transportation estimates. Rev. Mat. Iberoamericana **19** (2003), no. 3, 971–1018. MR2053570 (2005a:35126)
4. Cercignani, C.: The Boltzmann equation and its applications. Applied Mathematical Sciences **67**. Springer-Verlag, New York, 1988. MR1313028 (95i:82082)
5. Cercignani, C., Illner, R. and Pulvirenti, M.: The Mathematical theory of dilute gases. Applied Mathematical Sciences **106**. Springer-Verlag, New York, 1994. MR1307620 (96g:82046)
6. Czechowski, Z. and Palczewski, A.: Spectrum of the Boltzmann col- lision operator for radial cut-off potentials. Bull. Acad. Polon. Sci. S'er. Sci. Tech. **28** (1980), no. 9–10, 387–396. MR0654555 (83e:76053)
7. Degond, P. and Lemou, M.: Dispersion relations for the linearized Fokker-Planck equation. Arch. Rational Mech. Anal. **138** (1997), no. 2, 137–167. MR1463805 (99f:82051)

8. Desvillettes, L.: Entropy dissipation rate and convergence in kinetic equations. Comm. Math. Phys. **123** (1989), no. 4, 687–702. MR1006301 (90g:82050)
9. Desvillettes, L.: On asymptotics of the Boltzmann equation when the collisions become grazing. Transport Theory Statist. Phys. **21** (1992), no. 3, 259–276. MR1165528 (93h:82051)
10. Desvillettes, L., Convergence towards the thermodynamical equilib- rium. In Trends in applications of mathematics to mechanics (Nice, 1998), 115–126. Monogr. Surv. Pure Appl. Math. **106**. Chapman & Hall/CRC, Boca Raton, FL, 2000. MR1734876
11. Desvillettes, L. and Villani, C: On the spatially homogeneous Lan- dau equation for hard potentials. II. *H* -theorem and applications. Comm. Partial Differential Equations **25** (2000), no. 1-2, 261–298. MR1737548 (2001c:82066)
12. Ellis, R. S. and Pinsky, M. A.: The first and second fluid approxima- tions to the linearized Boltzmann equation. J. Math. Pures Appl. (9) **54** (1975), 125–156. MR0609540 (58 #29430b)
13. Golse, F. and Poupaud, F.: Un r'esultat de compacit'e pour l"equation de Boltzmann avec potentiel mou. Application au probl'eme de demi-espace, C. R. Acad. Sci. Paris S'er. I Math. **303** (1986), no. 12, 583–586. MR0867945 (87m:35190)
14. Klaus, M.: Boltzmann collision operator without cut-off. Helv. Phys. Acta **50** (1977), no. 6, 893–903. MR0523550 (58 #25683)
15. Pao, Y. P.: Boltzmann collision operator with inverse-power intermolecu- lar potentials. I, II. Comm. Pure Appl. Math. **27** (1974), 407–428, 559–581. MR0636407 (58 #30519)
16. Villani, C.: A review of mathematical topics in collisional kinetic the- ory. In Handbook of mathematical fluid dynamics, Vol. I, 71–305. North- Holland, Amsterdam, 2002. MR1942465 (2003k:82087)
17. Villani, C.: On the spatially homogeneous Landau equation for Maxwellian molecules. Math. Models Methods Appl. Sci. **8** (1998), no. 6, 957–983. MR1646502 (99i:82036)
18. Villani, C.:, Contribution 'al"etude math'ematique des collisions en th'eorie cin'etique. Master's thesis, Univ. Paris–Dauphine France, 2000.
19. Wang Chang, C. S., Uhlenbeck, G. E. and De Boer, J.: Studies in Statistical Mechanics, Vol. V. North-Holland, Amsterdam, 1970.
20. Wennberg, B.: On an entropy dissipation inequality for the Boltzmann equation. C. R. Acad. Sci. Paris S'er. I Math. **315** (1992), no. 13, 1441–1446. MR1199019 (93j:82063)

Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.

MR2226800 92C50 (62E10 65C40 76Z99)

Baranger, C. (F-ENSET-AM); **Boudin, L.** (F-PARIS6-N); **Jabin, P.-E.** (F-NICE-LD);
Mancini, S. [Mancini, Simona] (F-ORLN-AM)

A modeling of biospray for the upper airways. (English, French summaries)

CEMRACS 2004—*mathematics and applications to biology and medicine*, 41–47 (*electronic*), *ESAIM Proc.*, 14, *EDP Sci.*, *Les Ulis*, 2005.

{This item will not be reviewed individually. For details of the collection in which this item appears see MR2229136 .}

{For the entire collection see MR2229136}

MR2202266 (2006j:81091) 81R12 (81-02 81T99)

Hélein, Frédéric (F-ENSET-AM); **Kouneiher, Joseph** (F-PARIS7)

On the soliton-particle dualities. (English summary)

Geometries of nature, *living systems and human cognition*, 93–128, *World Sci. Publ.*, *Hackensack*, *NJ*, 2005.

Summary: "In some field theories we have the striking feature that there are two different spectra: there is the spectrum of particle-like solitons and the spectrum of the actual particles. From this idea that in certain field theories the two spectra may be interrelated or interlinked particle-soliton duality emerges. This phenomenon is surprising and deep, occurs elsewhere in field theory and has had important applications in supersymmetric field theory and in superstring theory. It is similar to, and linked with, T-duality. In this paper we investigate the foundations and the development of this duality."

{For the entire collection see MR2194170 (2006h:00005)}

Reviewed by *H. S. Blas Achic* (Cuiabá)

MR2203948 (2006i:94010) 94A08 (49N45 62H12 62H35 94A12)
Nikolova, Mila (F-ENSET-AM)
Analysis of the recovery of edges in images and signals by minimizing nonconvex regularized least-squares. (English summary)
Multiscale Model. Simul. **4** (2005), *no. 3,* 960–991 (*electronic*).

The problem is to estimate an unknown image x from the measured data $y = Ax + n$. It is assumed that $x \in \mathbb{R}^p$, $n \in \mathbb{R}^q$ is noise and $A \in \mathbb{R}^{q\times p}$. The estimate minimizes

$$F_y(x) = \|Ax - y\|^2 + \beta\Phi(x).$$

The second term is a regularization term with $\beta > 0$ and $\Phi(x) = \sum_{i\in J} \varphi(g_i^\top x)$. The $g_i^\top$ represent difference operators (e.g., first order differences between neighboring pixels). The paper derives properties for the minimizer when the potential function φ is not convex. Depending on the properties of φ (especially the smoothness at 0) and the parameter β, the set J can be subdivided into two sets: one will enhance smoothness of the solution, the other will enhance the edges. Several cases are investigated and illustrated by numerical examples.

Reviewed by *A. Bultheel*

References

1. S. Alliney, *Digital filters as absolute norm regularizers*, IEEE Trans. Signal Process., 40 (1992), pp. 1548–1562.
2. S. Alliney, *A property of the minimum vectors of a regularizing functional defined by means of absolute norm*, IEEE Trans. Signal Process., 45 (1997), pp. 913–917.
3. G. Aubert and P. Kornprobst, *Mathematical Problems in Images Processing*, Springer- Verlag, Berlin, 2002. MR1865346 (2002m:49001)
4. J. E. Besag, *Digital image processing: Towards Bayesian image analysis*, J. Appl. Statist., 16 (1989), pp. 395–407.
5. A. Blake and A. Zisserman, *Visual Reconstruction*, MIT Press, Cambridge, MA, 1987. MR0919733 (89k:92086)
6. C. Bouman and K. Sauer, *A generalized Gaussian image model for edge-preserving* map *estimation*, IEEE Trans. Image Process., 2 (1993), pp. 296–310.
7. A. Chambolle, *Finite-differences discretizations of the Mumford-Shah functional*, M2AN Math. Model. Numer. Anal., 33 (1999), pp. 261–288. MR1700035 (2000d:49049)
8. T. F. Chan and S. Esedo$\overline{glu}$, *Aspects of total variation regularized $L1$ function approximation*, SIAM J. Appl. Math., 65 (2005), pp. 1817–1837. MR2177726 (2006k:94003)
9. P. Charbonnier, L. Blanc-F'eraud, G. Aubert, and M. Barlaud, *Deterministic edge- preserving regularization in computed imaging*, IEEE Trans. Image Process., 6 (1997), pp. 298–311.
10. G. Demoment, *Image reconstruction and restoration: Overview of common estimation structure and problems*, IEEE Trans. Acoust. Speech Signal Process., 37 (1989), pp. 2024–2036.
11. D. Donoho, I. Johnstone, J. Hoch, and A. Stern, *Maximum entropy and the nearly black object*, J. Roy. Statist. Soc. Ser. B, 54 (1992), pp. 41–81. MR1157714 (93d:62010)
12. S. Durand and M. Nikolova, *Stability of minimizers of regularized least squares objective*

functions I: *Study of the local behaviour*, Appl. Math. Optim., to appear. cf. MR 2006k:90132
13. S. Durand and M. Nikolova, *Stability of minimizers of regularized least squares objective functions* II: *Study of the global behaviour*, Appl. Math. Optim., to appear. cf. MR 2006m:49047
14. D. Geman, *Random fields and inverse problems in imaging*, in 'Ecole d' 'Et'e de Probabilit'es de Saint-Flour XVIII—1988, Lecture Notes in Math. 1427, Springer-Verlag, Berlin, 1990, pp. 113–193. MR1100283 (92f:68183)
15. D. Geman and G. Reynolds, *Constrained restoration and recovery of discontinuities*, IEEE Transactions on Pattern Analysis and Machine Intelligence, 14 (1992), pp. 367–383.
16. D. Geman and C. Yang, *Nonlinear image recovery with half-quadratic regularization*, IEEE Trans. Image Process., 4 (1995), pp. 932–946.
17. S. Geman and D. Geman, *Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images*, IEEE Transactions on Pattern Analysis and Machine Intelligence, 6 (1984), pp. 721–741.
18. S. Geman and D. E. McClure, *Statistical methods for tomographic image reconstruction*, Bull. Inst. Internat. Statist., 52 (1987), pp. 5–21. MR1027188
19. P. J. Green, *Bayesian reconstructions from emission tomography data using a modified* em *algorithm*, IEEE Transactions on Medical Imaging, 9 (1990), pp. 84–93.
20. T. Hebert and R. Leahy, *A generalized EM algorithm for* 3-*D Bayesian reconstruction from Poisson data using Gibbs priors*, IEEE Transactions on Medical Imaging, 8 (1989), pp. 194–202.
21. K. Lange, *Convergence of EM image reconstruction algorithms with Gibbs priors*, IEEE Transactions on Medical Imaging, 9 (1990), pp. 439–446.
22. Y. Leclerc, *Constructing simple stable description for image partitioning*, International Journal of Computer Vision, 3 (1989), pp. 73–102.
23. S. Li, *Markov Random Field Modeling in Computer Vision*, 1st ed., Springer-Verlag, New York, 1995.
24. J.-M. Morel and S. Solimini, *Segmentation of images by variational methods: A constructive approach*, Rev. Mat. Univ. Complut. Madrid, 1 (1988), pp. 169–182. MR0977048 (89k:90183)
25. J.-M. Morel and S. Solimini, *Variational Methods in Image Segmentation*, Birkh"auser, Basel, 1995. MR1321598 (96b:68184)
26. D. Mumford and J. Shah, *Boundary detection by minimizing functionals*, in Proceedings of the IEEE International Conference on Acoustics, Speech and Signal Processing, 1985, pp. 22–26.
27. D. Mumford and J. Shah, *Optimal approximations by piecewise smooth functions and associated variational problems*, Comm. Pure Appl. Math., 42 (1989), pp. 577–685. MR0997568 (90g:49033)
28. M. Nikolova, *Estim'ees localement fortement homog'enes*, C. R. Acad. Sci. Paris S'er. I Math., 325 (1997), pp. 665–670. MR1473843 (98d:62180)
29. M. Nikolova, *Markovian reconstruction using a GNC approach*, IEEE Trans. Image Process., 8 (1999), pp. 1204–1220.
30. M. Nikolova, *Local strong homogeneity of a regularized estimator*, SIAM J. Appl. Math., 61 (2000), pp. 633–658. MR1780806 (2001g:94005)
31. M. Nikolova, *Minimizers of cost-functions involving nonsmooth data-fidelity terms. Applica-*

tion to the processing of outliers, SIAM J. Numer. Anal., 40 (2002), pp. 965–994. MR1949401 (2003k:62170)
32. M. Nikolova, *A variational approach to remove outliers and impulse noise*, J. Math. Imaging Vision, 20 (2004), pp. 99–120. MR2049784 (2005b:94006)
33. M. Nikolova, *Weakly constrained minimization. Application to the estimation of images and signals involving constant regions*, J. Math. Imaging Vision, 21 (2004), pp. 155–175. MR2090130 (2005g:94020)
34. P. Perona and J. Malik, *Scale-space and edge detection using anisotropic diffusion*, IEEE Transactions on Pattern Analysis and Machine Intelligence, 12 (1990), pp. 629–639.
35. W. Ring, *Structural properties of solutions of total variation regularization problems*, M2AN Math. Model. Numer. Anal., 34 (2000), pp. 799–810. MR1784486 (2001g:65077)
36. L. Rudin, S. Osher, and C. Fatemi, *Nonlinear total variation based noise removal algorithm*, Phys. D, 60 (1992), pp. 259–268.
37. S. Saquib, C. Bouman, and K. Sauer, *ML parameter estimation for Markov random fields, with applications to Bayesian tomography*, IEEE Trans. Image Process., 7 (1998), pp. 1029–1044.
38. G. Steidl, J. Weickert, T. Brox, P. Mr'azek, and M. Welk, *On the equivalence of soft wavelet shrinkage, total variation diffusion, total variation regularization, and SIDEs*, SIAM J. Numer. Anal., 42 (2004), pp. 686–713. MR2084232 (2006d:94018)
39. R. Stevenson and E. Delp, *Fitting curves with discontinuities*, in Proceedings of the 1st International Workshop on Robust Computer Vision, Seattle, WA, 1990, pp. 127–136.
40. A. Tikhonov and V. Arsenin, *Solutions of Ill-Posed Problems*, Winston, Washington, DC, 1977. MR0455365 (56 #13604)
41. C. R. Vogel and M. E. Oman, *Iterative method for total variation denoising*, SIAM J. Sci. Comput., 17 (1996), pp. 227–238. MR1375276

Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.

MR2199915 (2007f:49035) 49N10 (35A15 65K10 94A08 94A12)
Nikolova, Mila (F-ENSET-AM); **Ng, Michael K.** (PRC-HK)

Analysis of half-quadratic minimization methods for signal and image recovery. (English summary)

SIAM J. Sci. Comput. **27** (2005), *no. 3,* 937–966 (*electronic*).

Summary: "We address the minimization of regularized convex cost functions which are customarily used for edge-preserving restoration and reconstruction of signals and images. In order to accelerate computation, the *multiplicative* and the *additive* half-quadratic reformulation of the

original cost-function have been pioneered in [D. Geman and G. Reynolds, IEEE Trans. Pattern Anal. Mach. Intell. **14** (1992), no. 3, 367–383; G. Geman and C. Yang, IEEE Trans. Image Process. **4** (1995), no. 7, 932–946]. The alternate minimization of the resultant (augmented) cost-functions has a simple explicit form. The goal of this paper is to provide a systematic analysis of the convergence rate achieved by these methods. For the multiplicative and additive half-quadratic regularizations, we determine their upper bounds for their root-convergence factors. The bound for the multiplicative form is seen to be always smaller than the bound for the additive form. Experiments show that the number of iterations required for convergence for the multiplicative form is always less than that for the additive form. However, the computational cost of each iteration is much higher for the multiplicative form than for the additive form. The global assessment is that minimization using the additive form of half-quadratic regularization is faster than using the multiplicative form. When the additive form is applicable, it is hence recommended. Extensive experiments demonstrate that in our MATLAB implementation, both methods are substantially faster (in terms of computational times) than the standard MATLAB Optimization Toolbox routines used in our comparison study."

References

1. R. Acar and C. Vogel, *Analysis of bounded variation penalty methods for ill-posed problems*, IEEE Trans. Image Process., 10 (1994), pp. 1217–1229. MR1306801 (95i:65092)
2. G. Aubert and P. Kornprobst, *Mathematical Problems in Image Processing*, Springer-Verlag, Berlin, 2002. MR1865346 (2002m:49001)
3. G. Aubert and L. Vese, *A variational method in image recovery*, SIAM J. Numer. Anal., 34 (1997), pp. 1948–1979. MR1472205 (99a:65182)
4. M. Black and A. Rangarajan, *On the unification of line processes, outlier rejection, and robust statistics with applications to early vision*, Internat. J. Computer Vision, 19 (1996), pp. 57–91.
5. C. Bouman and K. Sauer, *A generalized Gaussian image model for edge-preserving* map *estimation*, IEEE Trans. Image Process., 2 (1993), pp. 296–310.
6. R. H. Chan and M. K. Ng, *Conjugate gradient methods for Toeplitz systems*, SIAM Rev., 38 (1996), pp. 427–482. MR1409592 (97i:65048)
7. P. Charbonnier, L. Blanc-F'eraud, G. Aubert, and M. Barlaud, *Two deterministic half- quadratic regularization algorithms for computed imaging*, in Proceedings of IEEE ICIP, Vol. 2, 1994, pp. 168–172.
8. P. Charbonnier, L. Blanc-F'eraud, G. Aubert, and M. Barlaud, *Deterministic edge- preserving regularization in computed imaging*, IEEE Trans. Image Process., 6 (1997), pp. 298–311.
9. P. G. Ciarlet, *Introduction 'a l'analyse num'erique matricielle et 'a l'optimisation*, Collection math'ematiques appliqu'ees pour la maîtrise, Masson, Paris, 1990.
10. L. D. Cohen, *Auxiliary variables and two-step iterative algorithms in computer vision problems*, J. Math. Imaging Vision, 6 (1996), pp. 376–386. MR1379524 (97b:68231)
11. A. H. Delaney and Y. Bresler, *Globally convergent edge-preserving regularized reconstruction: An application to limited-angle tomography*, IEEE Trans. Image Process., 7 (1998), pp. 204–221.
12. G. Demoment, *Image reconstruction and restoration: Overview of common estimation struc-*

tures and problems, IEEE Trans. Acoustics Speech Signal Process., 37 (1989), pp. 2024– 2036.
13. D. Geman and G. Reynolds, *Constrained restoration and the recovery of discontinuities*, IEEE Trans. Pattern Anal. Machine Intelligence, 14 (1992), pp. 367–383.
14. D. Geman and C. Yang, *Nonlinear image recovery with half-quadratic regularization*, IEEE Trans. Image Process., 4 (1995), pp. 932–946.
15. S. Geman and D. Geman, *Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images*, IEEE Trans. Pattern Anal. Machine Intelligence, 6 (1984), pp. 721–741.
16. R. Glowinski, J. Lions, and R. Tr'emoli'eres, *Analyse num'erique des in'equations variationnelles*, Vol. 1, 1st ed., Dunod, Paris, 1976.
17. P. J. Green, *Bayesian reconstructions from emission tomography data using a modified* em *algorithm*, IEEE Trans. Medical Imaging, 9 (1990), pp. 84–93.
18. J.-B. Hiriart-Urruty and C. Lemar'echal, *Convex Analysis and Minimization Algorithms*, Vol. I, Springer-Verlag, Berlin, 1996.
19. J. Idier, *Convex half-quadratic criteria and interacting auxiliary variables for image restoration*, IEEE Trans. Image Process., 10 (2001), pp. 1001–1009. MR1838034 (2002e:94010)
20. T. Kailath and A. H. Sayed, *Displacement structure: Theory and applications*, SIAM Rev., 37 (1995), pp. 297–386. MR1355506 (96h:15015)
21. P. Kornprobst, R. Deriche, and G. Aubert, *Image sequence analysis via partial differential equations*, J. Math. Imaging Vision, 11 (1999), pp. 5–26. MR1715446 (2000k:94005)
22. S. Li, *Markov Random Field Modeling in Computer Vision*, 1st ed., Springer-Verlag, New York, 1995.
23. M. Ng, R. Chan, T. Chan, and A. Yip, *Cosine transform preconditioners for high resolution image reconstruction*, Linear Algebra Appl., 316 (2000), pp. 89–104. MR1782418 (2001e:94004)
24. M. K. Ng, R. H. Chan, and W.-C. Tang, *A fast algorithm for deblurring models with Neumann boundary conditions*, SIAM J. Sci. Comput., 21 (1999), pp. 851–866. MR1718798 (2000g:65025)
25. M. Nikolova, *Local strong homogeneity of a regularized estimator*, SIAM J. Appl. Math., 61 (2000), pp. 633–658. MR1780806 (2001g:94005)
26. J. Ortega and W. Rheinboldt, *Iterative Solution of Nonlinear Equations in Several Vari- ables*, Academic Press, New York, 1970. MR0273810 (42 #8686)
27. R. T. Rockafellar, *Convex Analysis*, Princeton University Press, Princeton, NJ, 1970. MR0274683 (43 #445)
28. L. Rudin, S. Osher, and E. Fatemi, *Nonlinear total variation based noise removal algorithms*, Phys. D, 60 (1992), pp. 259–268.
29. A. Tarantola, *Inverse Problem Theory: Methods for Data Fitting and Model Parameter Estimation*, Elsevier–North Holland, Amsterdam, 1987. MR0930881 (89b:65007)
30. S. Teboul, L. Blanc-F'eraud, G. Aubert, and M. Barlaud, *Variational approach for edge-preserving regularization using coupled PDE's*, IEEE Trans. Image Process., 7 (1998), pp. 387–397.
31. A. Tikhonov and V. Arsenin, *Solutions of Ill-Posed Problems*, Winston, Washington, DC, 1977. MR0455365 (56 #13604)

MR2186371 (2006f:76073) 76N99 (76D25 76M25 82D15)
Baranger, C. (F-ENSET-AM); **Baudin, G.** (F-ENSET-AM); **Boudin, L.** (F-PARIS6-N); **Després, B.** (F-PARIS6-N); **Lagoutière, F.** (F-PARIS6-N); **Lapébie, E.** (F-ENSET-AM); **Takahashi, T. [Takahashi, Takéo]** (F-NANC-IE)

Liquid jet generation and break-up. (English summary)

Numerical methods for hyperbolic and kinetic problems, 149–176, *IRMA Lect. Math. Theor. Phys.*, 7, *Eur. Math. Soc., Zürich*, 2005.

Summary: "This work is motivated by the numerical simulation of the generation and break-up of droplets after the impact of a rigid body on a tank filled with a compressible fluid. This paper splits into two very different parts. The first part deals with the modeling and the numerical resolution of a spray of liquid droplets in a compressible medium like air. Phenomena taken into account are the breakup effects due to the velocity and pressure waves in the compressible ambient fluid. The second part is concerned with the transport of a rigid body in a compressible liquid, involving reciprocal effects between the two components. A new one-dimensional algorithm working on a fixed Eulerian mesh is proposed."

{For the entire collection see MR2187100 (2006f:65005)}

MR2184882 (2006g:76018) 76B25 (76B07 76B15 76M25)
Binder, B. J. (4-EANG); **Vanden-Broeck, J.-M.** (4-EANG); **Dias, F. [Dias, Frédéric]** (F-ENSET-AM)

Forced solitary waves and fronts past submerged obstacles. (English summary)

Chaos **15** (2005), *no. 3,* 037106, 13 *pp.*

Summary: "Herein, an efficient numerical method is presented to describe the flow of a liquid in an open channel with various types of bottom configurations. The method is developed for steady two-dimensional potential free surface flows. The resulting nonlinear problem is solved numerically

by boundary integral equation methods. In addition weakly nonlinear solutions are derived. New solutions which complement those of F. Dias and J.-M. Vanden-Broeck [J. Fluid Mech. **509** (2004), 92–102; Zbl 1060.76019] are presented. Furthermore some solutions for channel flows past dips in the bottom are discussed."

MR2182135 (2006j:65360) 65N30 (49M25 58E20 65N50)

Pierre, Morgan (F-ENSET-AM)

Weak BV convergence of a moving finite-element method for singular axisymmetric harmonic maps. (English summary)

SIAM J. Numer. Anal. **43** (2005), *no. 4,* 1436–1454 (*electronic*).

The author proves the convergence of an optimal mesh method for the midpoint formula in the presence of a consistency error and establishes an external approximation by BV functions. The solution of the continuous problem at hand minimizes a relaxed Dirichlet energy among axisymmetric maps from the disk to the sphere. It is defined on $[0, 1]$ and has a boundary layer of zero thickness at 0. Because of the consistency error introduced by the discretization of the energy, the discrete minimizer is nonconforming. The main difficulty is to find appropriate error estimators.

Reviewed by *Rémi Vaillancourt*

References

1. F. Alouges, *A new algorithm for computing liquid crystal stable configurations: The harmonic mapping case*, SIAM J. Numer. Anal., 34 (1997), pp. 1708–1726. MR1472192 (98k:82190)
2. F. Alouges and M. Pierre, *Mesh optimization for singular axisymmetric harmonic maps from the disc into the sphere*, Numer. Math., to appear. cf. MR 2006i:65216
3. N. Carlson and K. Miller, *Design and application of a gradient-weighted moving finite element code* I: *In one dimension*, SIAM J. Sci. Comput., 19 (1998), pp. 728–765. MR1616670 (99e:65130)
4. P. de Oliveira, *Existence de maillages optimaux dans les m'ethodes d"el'ements finis*, RAIRO Anal. Num'er., 14 (1980), pp. 279–290. MR0592754 (83c:65250)
5. M. Delfour, G. Payre, and J. P. Zolesio, *An optimal triangulation for second-order elliptic problems*, Comput. Methods Appl. Mech. Engrg., 50 (1985), pp. 231–261. MR0800331 (86j:65151)
6. I. Ekeland and R. Temam, *Convex Analysis and Variational Problems*, North-Holland, Amsterdam, 1976. MR0463994 (57 #3931b)
7. M. Giaquinta, G. Modica, and J. Souček, *Cartesian Currents in the Calculus of Variations* I,

Springer-Verlag, Berlin, 1998. MR1645086 (2000b:49001a)

8. M. Giaquinta, G. Modica, and J. Souček, *Cartesian Currents in the Calculus of Varia- tions* II, Springer-Verlag, Berlin, 1998. MR1645086 (2000b:49001a)
9. F. H”ulsemann and Y. Tourigny, *A new moving mesh algorithm for the finite element solution of variational problems*, SIAM J. Numer. Anal., 35 (1998), pp. 1416–1438. MR1620164 (99f:65178)
10. R. Li, W.-B. Liu, and T. Tang, *Adaptive finite element approximation for distributed elliptic optimal control problems*, SIAM J. Control Optim., 41 (2002), pp. 1321–1349. MR1971952 (2004a:49036)
11. R. Li, T. Tang, and P.-W. Zhang, *Moving mesh methods in multiple dimensions based on harmonic maps*, J. Comput. Phys., 170 (2001), pp. 562–588. MR1844903 (2002e:65139)
12. R. Li, T. Tang, and P.-W. Zhang, *A moving mesh finite element algorithm for singular problems in two and three space dimensions*, J. Comput. Phys., 177 (2002), pp. 365–393. MR1897293 (2003c:65093)
13. M. Pierre, *Graphes et Maillages Adapt’es Pour le Calcul d’Applications Harmoniques Minimisantes*, Ph.D. thesis, ENS Cachan, France, 2002.
14. A. Quarteroni, R. Sacco, and F. Saleri, *Numerical Mathematics*, Springer-Verlag, New- York, 2000. MR1751750 (2000m:65001)
15. S. Rippa and B. Schiff, *Minimum energy triangulations for elliptic problems*, Comput. Methods Appl. Mech. Engrg., 84 (1990), pp. 257–274. MR1082824 (91m:65277)
16. A. Soyeur, *The Dirichlet problem for harmonic maps from the disc into the* 2*-sphere*, Proc. Roy. Soc. Edinburgh Sect. A., 113 (1989), pp. 229–234. MR1037728 (90m:58050)
17. T. Tang, *Moving mesh methods for computational fluid dynamics*, in Recent Advances in Adaptive Computations, Z.-C. Shi, Z. Chen, T. Tang, and D. Yu, eds., Contemp. Math., AMS, Providence, RI, 2005, to appear. cf. MR 2006j:76099

Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.

MR2178065 (2006g:35091) 35J70 (35K65 49J40 52A20)

Alter, F. (F-ENSET-AM); **Caselles, V.** (E-POFA-T); **Chambolle, A.** (F-PARIS9-A)

A characterization of convex calibrable sets in $\mathbb{R}^N$. (English summary)

Math. Ann. **332** (2005), *no. 2,* 329–366.

The authors give a characterization of the calibrability of bounded convex sets in $\mathbb{R}^N$ in terms of the mean curvature of the boundary, extending the known result in the case of two dimensions. Several corollaries are obtained, including that any bounded convex C of class $C^{1,1}$ contains a calibrable set K in its interior and that for any volume V, $V \in [|K|, |C|]$, the solution of the perimeter minimizing problem with fixed volume V in the class of sets contained in C is a convex set. Applications to the capillary problem in absence of gravity, to the minimizing total variation flow as well as to the eigenvalue problem for the p-Laplacian with $p = 1$ are also given.

Reviewed by *Jesús Hernández*

References

1. Alter, F., Caselles, V., Chambolle, A.: Evolution of Convex Sets in the Plane by the Minimizing Total Variation Flow. Interfaces Free Boundaries April 2005. MR2126142 (2006b:35154)
2. Ambrosio, L.: Corso introduttivo alla teoria geometrica della misura ed alle supefici minime. Scuola Normale Superiore, Pisa, 1997 MR1736268 (2000k:49001)
3. Ambrosio, L.: Minimizing movements. Rend. Accad. Naz. Sci. XL Mem. Mat. Appl. (5) **19**, 191–246 (1995) MR1387558 (97c:49044)
4. Ambrosio, L., Fusco, N., Pallara, D.: Functions of Bounded Variation and Free Discontinuity Problems. Oxford Mathematical Monographs, 2000 MR1857292 (2003a:49002)
5. Andreu, F., Ballester, C., Caselles, V., Mazón, J.M.: Minimizing total variational flow. Differential Integral Equations **4**(3), 321–360 (2001) MR1799898 (2002e:35109)
6. Andreu, F., Ballester, C., Caselles, V., Mazón, J.M.: The Dirichlet Problem for the Total Variation Flow. J. Funct. Anal. **180**, 347–403 (2001) MR1814993 (2001k:35143)
7. Andreu, F., Caselles, V., Mazón, J.M.: A parabolic quasilinear problem for linear growth functionals. Rev. Mat. Iberoamericana **18**, 135–185 (2002) MR1924690 (2003j:35142)
8. Anzellotti, G.: Pairings between measures and bounded functions and compensated compactness. Ann. Mat. Pura Appl. **135**, 293–318 (1983) MR0750538 (85m:46042)
9. Atkinson, F.V., Peletier, L.A., Bounds for vertical points of solutions of prescribed mean curvature type equations. Proc. Roy. Soc. Edinburgh Sect. A **112**, 15–32 (1989) MR1007535 (90m:35059)
10. Barozzi, E.: The curvature of a boundary with finite area. Atti Accad. Naz. Lincei Cl. Sci. Fis. Mat. Natur. Rend. Lincei (9) Mat. Appl. **5**, 149–159 (1994) MR1292570 (95h:49067)
11. Bellettini, G., Caselles, V., Novaga, M.: The Total Variation Flow in IR^N. J. Differential Equations **184**, 475–525 (2002) MR1929886 (2003g:35105)
12. Bellettini, G., Caselles, V., Novaga, M.: Explicit solutions of the eigenvalue problem $-\mathrm{div}\frac{Du}{|Du|} = u$. Siam J. Mathematical Analysis, 2005.
13. Bellettini, G., Novaga, M., Paolini, M.: On a crystalline variational problem. II. BV regularity and structure of minimizers on facets. Arch. Ration. Mech. Anal. **157**(3), 193–217 (2001) MR1826965 (2002c:49072b)
14. Bellettini, G., Novaga, M., Paolini, M.: Characterization of facet-breaking for nonsmooth mean curvature flow in the convex case. Interfaces Free Bound. **3**, 415–446 (2001) MR1869587 (2002k:53127)
15. Brezis, H.: Operateurs Maximaux Monotones. North Holland, 1973 MR0348562 (50 #1060)
16. Chambolle, A.: An algorithm for total variation minimization and applications. J. Math. Imaging Vision **20**, 89–97 (2004) MR2049783 (2005m:49058)
17. Chambolle, A.: An algorithm for mean curvature motion. Interfaces Free Bound. 6(2). 195–218

(2004). MR2079603 (2005f:53106)
18. Chen, J.T.: On the existence of capillary free surfaces in the absence of gravity. Pacific J. Math. **88**, 323–361 (1980) MR0607983 (82g:49037)
19. Concus, P., Finn, R.: On Capillary Free Surfaces in the Absence of Gravity. Acta Math. **132**, 177–198 (1974) MR0670441 (58 #32327a)
20. Crandall, M.G., Liggett, T.M.: Generation of semigroups of nonlinear transformations on general Banach spaces. Am. J. Math. **93**, 265–298 (1971) MR0287357 (44 #4563)
21. Evans, L.C., Gariepy, R.F.: Measure Theory and Fine Properties of Functions. Studies in Advanced Math. CRC Press, Ann Harbor, 1992 MR1158660 (93f:28001)
22. Finn, R.: Existence and non existence of capillary surfaces. Manuscripta Math. **28**, 1–11 (1979) MR0535691 (80f:58044a)
23. Finn, R.: A subsidiary variational problem and existence criteria for capillary surfaces. J. Reine Angew. Math. **353**, 196–214 (1984) MR0765833 (86h:49046)
24. Finn, R.: Equilibrium Capillary Surfaces. Springer Verlag, 1986 MR0816345 (88f:49001)
25. Giga, M.H., Giga, Y., Kobayashi, R.: Very singular diffusion equations. Proc. of Taniguchi Conf. on Math. Advanced Studies in Pure Mathematics **31**, 93–125 (2001) MR1865089 (2002i:35099)
26. Giga, Y.: Singular diffusivity-facets, shocks and more. Hokkaido University Preprint Series in Mathematics, Series 604, September 2003 cf. MR2296265
27. Giusti, E.: On the equation of surfaces of prescribed mean curvature. Existence and uniqueness without boundary conditions. Invent. Math. **46**, 111–137 (1978) MR0487722 (58 #7337)
28. Giusti, E.: Boundary Value Problems for Non-Parametric Surfaces of Prescribed Mean Curvature. Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) **3**, 501–548 (1976) MR0482506 (58 #2572)
29. Gonzalez, E., Massari, U.: Variational Mean Curvatures. Rend. Sem. Mat. Univ. Politec. Torino **52**, 1–28 (1994) MR1289900 (95e:49044)
30. Korevaar, N.: Capillary surface convexity above convex domains. Indiana Univ. Math. J. **32**, 73–82 (1983) MR0684757 (84b:35054)
31. Korevaar, N., Simon, L.: Equations of mean curvature type with contact angle boundary conditions. Geometric analysis and the calculus of variations, Internat. Press, Cambridge, MA, 1996, pp. 175–201 MR1449407 (98e:35071)
32. Lichnewski, A., Temam, R.: Pseudosolutions of the Time Dependent Minimal Surface Problem. J. Differential Equations **30**, 340–364 (1978) MR0521858 (80a:35059)
33. Massari, U.: Frontiere orientate di curvatura media assegnata in L^p. Rend. Sem. Mat. Univ. Padova **53**, 37–52 (1975) MR0417905 (54 #5953)
34. Meyer, Y.: Oscillating patterns in image processing and nonlinear evolution equations. The fifteenth Dean Jacqueline B. Lewis memorial lectures. University Lecture Series, 22. American Mathematical Society, Providence, RI, 2001 MR1852741 (2002j:43001)
35. Miranda, M.: Un principio di massimo forte per le frontiere minimali e una sua applicazione alla risoluzione del problema al contorno per l'equazione delle superfici di area minima. Rend. Sem. Mat. Univ. Padova **45**, 355–366 (1971) MR0303390 (46 #2527)
36. Rosales, C.: Isoperimetric regions in rotationally symmetric convex bodies. Indiana Univ. Math. J. **52**, 1201–1214 (2003) MR2010323 (2004h:58018)

37. Rudin, L., Osher, S., Fatemi, E.: Nonlinear total variation based noise removal algorithms. Physica D **60**, 259–268 (1992)
38. Santaló, L.A.: Integral geometry and geometric probability. Encyclopedia of Mathematics and its Applications, Vol. **1.** Addison-Wesley Publishing Co., Reading, Mass.-London-Amsterdam, 1976 MR0433364 (55 #6340)
39. Schneider, R.: Convex Bodies: The Brunn-Minkowski Theory. Encyclopedia of Mathematics and its Applications, 44, Cambridge University Press, 1993 MR1216521 (94d:52007)
40. Simon, L., Spruck, J.: Existence and Regularity of a Capillary Surface with Prescribed Contact Angle. Arch. Ration. Mech. Anal. **61**, 19–34 (1976) MR0487724 (58 #7339)
41. Stredulinsky, E., Ziemer, W.P.: Area Minimizing Sets Subject to a Volume Constraint in a Convex Set. J. Geom. Anal. **7**, 653–677 (1997) MR1669207 (99k:49089)
42. Ziemer, W.P.: Weakly Differentiable Functions. Springer Verlag, Ann Harbor, 1989 MR1014685 (91e:46046)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2177881 (2006g:65126) 65M06 (76M12)
Bouche, Daniel; **Ghidaglia, Jean-Michel** (F-ENSET-AM); **Pascal, Frédéric** (F-PARIS11-M)
Error estimate and the geometric corrector for the upwind finite volume method applied to the linear advection equation. (English summary)
SIAM J. Numer. Anal. **43** (2005), *no. 2,* 578–603 (*electronic*).

The authors address error estimation for convergence of finite volume-based upwind schemes applied to linear advection equations. Specifically, they consider solving the equation on a bounded domain with natural boundary conditions.

They analyze the problem of obtaining the optimal error estimate in the case of a general non-uniform grid, whereas there appears a loss of consistence for the standard convergence (Lax) proof. The introduction of this paper contains a wide state-of-the-art bibliography that a reader can find very useful.

The authors state in this paper that, differently from the answers provided in other studies, they are able to assess that, provided the solution is smooth, the error estimate is first order. For reaching this goal, they introduce a so-called geometric corrector with bounded norm, the mesh size constituting the bound.

Results are illustrated for some types of two-dimensional irregular grids generally obtained by means of triangulation.

Although in some ways limited in the analysis by the choice of using a simple model equation, the paper addresses an interesting issue sometimes not carefully considered in FVM computations on irregular grids. Hopefully, the authors will produce a further study for the variable velocity case.

Reviewed by *Filippo Maria Denaro*

References

1. C. Bardos, *Probl‘emes aux limites pour les ’equations aux d’eriv’ees partielles du premier ordre ‘a coefficients r’eels; th’eor‘emes d’approximation; application ‘a l”equation de transport*, Ann. Sci. ’Ecole Norm. Sup. (4), 3 (1970), pp. 185–233. MR0274925 (43 #683)
2. M. J. Berger, C. Helzel, and R. J. LeVeque, *h-box methods for the approximation of hyperbolic conservation laws on irregular grids*, SIAM J. Numer. Anal., 41 (2003), pp. 893– 918. MR2005187 (2004g:65103)
3. M. J. Berger, R. J. LeVeque, and L. G. Stern, *Finite volume methods for irregular one- dimensional grids*, in Mathematics of Computation 1943–1993: A Half-century of Com- putational Mathematics (Vancouver, BC, 1993), Proc. Sympos. Appl. Math. 48, AMS, Providence, RI, 1994, pp. 255–259. MR1314852
4. B. Cockburn and P.-A. Gremaud, *A priori error estimates for numerical methods for scalar conservation laws.* I: *The general approach*, Math. Comput., 65 (1996), pp. 533–573. MR1333308 (96g:65089)
5. B. Cockburn and P.-A. Gremaud, *A priori error estimates for numerical methods for scalar conservation laws.* II: *Flux-splitting monotone schemes on irregular Cartesian grids*, Math. Comput., 66 (1997), pp. 547–572. MR1408372 (97m:65173)
6. B. Cockburn, P.-A. Gremaud, and J. X. Yang, *A priori error estimates for numerical methods for scalar conservation laws. Part* III: *Multidimensional flux-splitting monotone schemes on non-Cartesian grids*, SIAM J. Numer. Anal., 35 (1998), pp. 1775–1803. MR1640021 (99g:65097)
7. B. Despr’es, *Th’eor‘eme de lax et volumes finis*, in Proceedings du 32e CANUM, La Grande-Motte, Universit’e de Montpellier et S.M.A.I., 2003.
8. B. Despr’es, *An explicit a priori estimate for a finite volume approximation of linear advection on non-Cartesian grids*, SIAM J. Numer. Anal., 42 (2004), pp. 484–504. MR2084223 (2005j:65095)
9. B. Despr’es, *Lax theorem and finite volume schemes*, Math. Comp., 73 (2004), pp. 1203–1234. MR2047085 (2005a:65083)
10. R. Eymard, T. Gallou”et, and R. Herbin, *Finite volume methods*, in Handbook of Numerical Analysis, P.-A. Ciarlet and J.-L. Lions, eds., Handb. Numer. Anal. 4, North-Holland, Amsterdam, 2000, pp. 713–1020. MR1804748 (2002e:65138)
11. B. Garcia-Archilla and J. M. Sanz-Serna, *A finite difference formula for the discretization of d3/dx3 on nonuniform grids*, Math. Comp., 57 (1991), pp. 239–257. MR1079016 (91j:65143)
12. C. Geuzaine and J.-F. Remacle, *Gmsh: A three-dimensional finite element mesh generator with built-in pre- and post-processing facilities*; available online from http://www.geuz.org/gmsh/, 1997–2005.
13. E. Godlewski and P.A. Raviart, *Numerical Approximation of Hyperbolic Systems of Conservation Laws*, Springer-Verlag, New York, 1996. MR1410987 (98d:65109)

14. J. D. Hoffman, *Relationship between the truncation errors of centered finite-difference approximations on uniform and nonuniform meshes*, J. Comput. Phys., 46 (1982), pp. 469–474. MR0673711 (83j:65031)
15. H.-O. Kreiss, T.A. Manteuffel, B. Swartz, B. Wendroff, and A. B. White, Jr., *Supra- convergent schemes on irregular grids*, Math. Comp., 47 (1986), 537–554. MR0856701 (88b:65082)
16. D. Kr”oner, *Numerical Schemes for Conservation Laws*, Wiley-Teubner Ser. Adv. Numer. Math., Wiley, Chichester, UK, 1997. MR1437144 (98b:65003)
17. N. N. Kuznetsov, *Accuracy of some approximate methods for computing the weak solutions of a first-order quasi-linear equation*, U.S.S.R. Comput. Math. Math. Phys., 16 (1976), pp. 105–119.
18. C. D. Levermore, T. A. Manteuffel, and A. B. White, Jr., *Numerical solution of partial differential equations on irregular grids*, in Computational Techniques and Applications: CTAC-87 (Sydney, 1987), North-Holland, Amsterdam, 1988, pp. 417–426. MR0951453 (90f:65148)
19. T. A. Manteuffel and A. B. White, Jr., *The numerical solution of second order boundary value problems on nonuniform meshes*, Math. Comput., 47 (1986), pp. 511–535. MR0856700 (87m:65116)
20. T. A. Manteuffel and A. B. White, Jr., *On the efficient numerical solution of systems of second order boundary value problems*, SIAM J. Numer. Anal., 23 (1986), pp. 996–1006. MR0859011 (87k:65100)
21. T. A. Manteuffel and A. B. White, Jr., *A calculus of difference schemes for the solution of boundary-value problems on irregular meshes*, SIAM J. Numer. Anal., 29 (1992), pp. 1321–1346. MR1182733 (94b:65139)
22. P. Monk and E. S”uli, *A convergence analysis of Yee’s scheme on nonuniform grids*, SIAM J. Numer. Anal., 31 (1994), pp. 393–412. MR1276707 (95a:65191)
23. T. E. Peterson, *A note on the convergence of the discontinuous Galerkin method for a scalar hyperbolic equation*, SIAM J. Numer. Anal., 28 (1991), pp. 133–140. MR1083327 (91m:65250)
24. J. Pike, *Grid adaptive algorithms for the solution of the Euler equations on irregular grids*, J. Comput. Phys., 71 (1987), pp. 194–223. MR0895526 (88g:65083)
25. Z.-H. Teng and P. Zhang, *Optimal $L1$-rate of convergence for the viscosity method and monotone scheme to piecewise constant solutions with shocks*, SIAM J. Numer. Anal., 34 (1997), pp. 959–978. MR1451109 (98f:65094)
26. A. N. Tikhonov and A. A. Samarskij, *Homogeneous difference schemes on non-uniform nets*, U.S.S.R. Comput. Math. Math. Phys., 3 (1964), pp. 927–953.
27. E. Turkel, *Accuracy of schemes with nonuniform meshes for compressible fluid flows*, Appl. Numer. Math., 2 (1986), pp. 529–550. MR0871091 (88m:65137)
28. J.-P. Vila and P. Villedieu, *Convergence of an explicit finite volume scheme for first order symmetric systems*, Numer. Math., 94 (2003), pp. 573–602. MR1981168 (2004d:65102)
29. B. Wendroff, *Supraconvergence in Two Dimensions*, Technical report LA-UR 95-3068, Los Alamos National Laboratory, Los Alamos, NM, 1995.
30. B. Wendroff and A. B. White, Jr., *Some supraconvergent schemes for hyperbolic equations on irregular grids*, in Nonlinear Hyperbolic Equations—Theory, Computation Methods, and Applications (Aachen, 1988), Notes Numer. Fluid Mech. 24, Vieweg, Braunschweig, 1989, pp.

671–677. MR0991404
31. B. Wendroff and A. B. White, Jr., *A supraconvergent scheme for nonlinear hyperbolic systems*, Comput. Math. Appl., 18 (1989), pp. 761–767. MR1009864 (90g:65121)

Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.

MR2177636 (2006g:90020) 90B20 (49Q20)
Bernot, Marc (F-ENSET-AM); **Caselles, Vicent** (E-POFA-T);
Morel, Jean-Michel (F-ENSET-AM)
Traffic plans. (English summary)
Publ. Mat. **49** (2005), *no. 2,* 417–451.

Summary: “In recent research in the optimization of transportation networks, a problem was formalized as finding the optimal paths to transport a measure μ^+ onto a measure μ^- with the same mass. This approach is realistic for simple good distribution networks (water, electric power, ...) but it is no longer realistic when we want to specify ‘who goes where’, like in the mailing problem or the optimal urban traffic network problem. In this paper, we present a new framework generalizing the former approaches and permitting solution of the optimal transport problem under the ‘who goes where’ constraint. This constraint is formalized as a transference plan from μ^+ to μ^- which we handle as a boundary condition for the ‘optimal traffic problem’.”

Reviewed by *Wilfrid Gangbo*

References

1. L. Ambrosio, N. Fusco and D. Pallara, ”*Functions of bounded variation and free discontinuity problems*”, Oxford Mathematical Monographs, The Clarendon Press, Oxford University Press, New York, 2000. MR1857292 (2003a:49002)
2. M. Bernot, V. Caselles and J.-M. Morel, Are there infinite irrigation trees?, Accepted in *J. Math. Fluid Mech.* MR2258415
3. S. Bhaskaran and F. J. M. Salzborn, Optimal design of gas pipeline networks, *J. Oper. Res. Soc.* **30** (1979), 1047–1060.
4. G. Buttazzo, Three optimization problems in mass transportation theory, Preprint, Dipartimento di Matematica, Università di Pisa, Pisa (2004). cf. MR 2006k:49113
5. G. Buttazzo and E. Stepanov, Optimal transportation networks as free Dirichlet regions for the Monge-Kantorovich problem, *Ann. Sc. Norm. Super. Pisa Cl. Sci. (5)* **2(4)** (2003), 631–678. MR2040639 (2004i:49080)
6. G. Buttazzo and E. Stepanov, Minimization problems for average distance functionals, in:

”*Calculus of variations: topics from the mathematical heritage of E. De Giorgi*”, Quad. Mat. **14**, Dept. Math., Seconda Univ. Napoli, Caserta, 2004, pp. 47–83. MR2118415 (2006b:49089)

7. V. Caselles and J.-M. Morel, Irrigation, in: ”*Variational methods for discontinuous structures*” (International Workshop in Villa Erba, Cernobbio, Italy, July 2001), Progress in Nonlinear Differential Equations and Their Applications **51** (Dal Maso, Gianni and Tomarelli, Franco, eds.), Birkhäuser, 2002, pp. 81–90. MR2197839
8. B. Dacorogna, ”*Direct methods in the calculus of variations*”, Applied Mathematical Sciences **78**, Springer-Verlag, Berlin, 1989. MR0990890 (90e:49001)
9. G. David and S. Semmes, ”*Fractured fractals and broken dreams*”, Self-similar geometry through metric and measure, Oxford Lecture Series in Mathematics and its Applications **7**, The Clarendon Press, Oxford University Press, New York, 1997. MR1616732 (99h:28018)
10. J. Diestel and J. J. Uhl, Jr., ”*Vector measures*”, With a foreword by B. J. Pettis, Mathematical Surveys **15**, American Mathematical Society, Providence, R.I., 1977. MR0453964 (56 #12216)
11. E. N. Gilbert, Minimum cost communication networks, *Bell System Tech. J.* **46**, (1967), 2209–2227.
12. E. N. Gilbert and H. O. Pollak, Steiner minimal trees, *SIAM J. Appl. Math.* **16** (1968), 1–29. MR0223269 (36 #6317)
13. L. Kantorovitch, On the translocation of masses, *C. R. (Doklady) Acad. Sci. URSS (N.S.)* **37** (1942), 199–201. MR0009619 (5,174d)
14. D. H. Lee, Low cost drainage networks, *Networks* **6** (1976), 351–371.
15. F. Maddalena, S. Solimini and J.-M. Morel, A variational model of irrigation patterns, *Interfaces Free Bound.* **5(4)** (2003), 391–415. MR2031464 (2004j:49065)
16. G. Monge, Mémoire sur la théorie des déblais et de remblais, Histoire de l'Académie Royale des Sciences de Paris (1781), 666–704.
17. C. Villani, ”*Topics in optimal transportation*”, Graduate Studies in Mathematics **58**, American Mathematical Society, Providence, RI, 2003. MR1964483 (2004e:90003)
18. D. Williams, ”*Probability with martingales*”, Cambridge Mathematical Textbooks, Cambridge University Press, Cambridge, 1991. MR1155402 (93d:60002)
19. Q. Xia, Optimal paths related to transport problems, *Commun. Contemp. Math.* **5(2)** (2003), 251–279. MR1966259 (2004a:90006)
20. Q. Xia, Interior regularity of optimal transport paths, *Calc. Var. Partial Differential Equations* **20(3)** (2004), 283–299. MR2062945 (2005a:90017)
21. G. Xue, T. P. Lillys and D. E. Dougherty, Computing the minimum cost pipe network interconnecting one sink and many sources, *SIAM J. Optim.* **10(1)** (1999), 22–42 (electronic). MR1722072 (2000h:90098)
22. J. Zhang and D. Zhu, A bilevel programming method for pipe network optimization, *SIAM J. Optim.* **6(3)** (1996), 838–857. MR1402208 (97b:90118)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2176922 (2006g:58010) 58B10 (49J45 68T10)
Trouvé, Alain (F-PARIS13-GA); **Younes, Laurent** (F-ENSET-AM)
Local geometry of deformable templates. (English summary)
SIAM J. Math. Anal. **37** (2005), *no. 1,* 17–59 (*electronic*).

The authors discuss a geometrical model of a space of deformable images or shapes where infinitesimal variations are combinations of elastic deformations (warping) and of photometric variations. Geodesics in such a space are related to velocity-based image warping methods that have proved to yield robust and efficient estimations of diffeomorphisms in the case of large deformations. A rigorous and general construction of such infinite-dimensional "shape manifolds" is provided, together with a Riemannian metric placed on it. Geodesic equations are then derived for which existence and uniqueness for all times are proved. This is used to deduce a geometrically founded linear approximation of the deformation of shapes in the neighborhood of a given template.

Reviewed by *Riccardo De Arcangelis*

References

1. Y. Amit, U. Grenander, and M. Piccioni, *Structural image restoration through deformable templates*, J. Amer. Statist. Assoc., 86 (1989), pp. 376–387.
2. Y. Amit and P. Piccioni, *A non-homogeneous Markov process for the estimation of Gaussian random fields with nonlinear observations*, Ann. Probab., 19 (1991), pp. 1664–1678. MR1127720 (92i:60095)
3. I. Arnold, *Mathematical Methods of Classical Mechanics*, Springer-Verlag, New York, 1989. MR0997295 (90c:58046)
4. V. Arnold and B. Khesin, *Topological methods in hydrodynamics*, in Annual Review Fluid of Mechanics, 24, Annual Review, Palo Alto, CA, 1992, pp. 145–166. MR1145009 (93d:58020)
5. V. I. Arnold, *Sur la g'eometrie differentielle des groupes de Lie de dimension infinie et ses applications 'a l'hydrodynamique des fluides parfaits*, Ann. Inst. Fourier (Grenoble), 1 (1966), pp. 319–361. MR0202082 (34 #1956)
6. R. Bajcsy and C. Broit, *Matching of deformed images*, in Proceedings of the 6th International Conference on Pattern Recognition, Munich, Germany, 1982, pp. 351–353.
7. R. Bajcsy and S. Kovacic, *Multiresolution elastic matching*, Comp. Vision, Graphics, and Image Proc., 46 (1989), pp. 1–21.
8. F. L. Bookstein, *Principal warps: Thin plate splines and the decomposition of deformations*, IEEE Trans. Pattern Anal. Mach. Intell., 11 (1989), pp. 567–585.
9. F. L. Bookstein, *Morphometric Tools for Landmark Data; Geometry and Biology*, Cambridge University Press, Cambridge, UK, 1991. MR1469220 (99d:92003)
10. H. Brezis, *Analyse Fonctionnelle, Th'eorie et Applications*, Masson, Paris, 1983. MR0697382 (85a:46001)

11. R. Camassa and D. D. Holm, *An integrable shallow water equation with peaked solitons*, Phys. Rev. Lett., 71 (1993), pp. 1661–1664. MR1234453 (94f:35121)
12. M. P. D. Carmo, *Riemannian Geometry*, Birkha"user, Basel, 1992. MR1138207 (92i:53001)
13. E. Christensen, R. D. Rabbitt, and M. I. Miller, *Deformable templates using large defor- mation kinematics*, IEEE Trans. Image Process., 1996, pp. 1435–1447.
14. T. Cootes, C. Taylor, D. Cooper, and J. Graham, *Active shape models: Their training and application*, Comp. Vis. and Image Understanding, 61 (1995), pp. 38–59.
15. C. David and S. W. Zucker, *Potentials, valleys, and dynamic global coverings*, Int. J. of Comp. Vision, 5 (1990), pp. 219–238.
16. P. Dupuis, U. Grenander, and M. Miller, *Variational problems on flows of diffeomorphisms for image matching*, Quart. Appl. Math., 56 (1998), pp. 587–600. MR1632326 (99f:49006)
17. C. Foias, D. D. Holm, and E. S. Titi, *The Navier-Stokes-alpha model of fluid turbulence*, Phys. D, 152 (2001), pp. 505–519. MR1837927 (2003d:76077)
18. D. Geman and S. Geman, *Stochastic relaxation, gibbs distribution and bayesian restoration of images*, IEEE Trans. Pattern Anal. Mach. Intell., 6 (1984), pp. 721–741.
19. U. Grenander, *Lectures in Pattern Theory*, Appl. Math. Sci. 33, Springer-Verlag, New York, 1981. MR0619018 (83c:68108)
20. U. Grenander, *General Pattern Theory*, Oxford Science Publications, Oxford University Press, New York, 1993. MR1270904 (96e:68118)
21. U. Grenander and D. M. Keenan, *Towards automated image understanding*, J. Appl. Stat., 16 (1989), pp. 207–221.
22. U. Grenander and D. M. Keenan, *On the shape of plane images*, SIAM J. Appl. Math., 53 (1993), pp. 1072–1094. MR1232168 (94c:60005)
23. U. Grenander and M. I. Miller, *Representations of knowledge in complex systems (with discussion section)*, J. Roy. Statist. Soc. Ser. B, 56 (1994), pp. 549–603. MR1293234 (96e:62003)
24. J. E. Marsden and T. S. Ratiu, *Introduction to Mechanics and Symmetry*, Springer-Verlag, New York, 1994. MR1304682 (95i:58073)
25. J. E. Marsden, T. Ratiu, and A. Weinstein, *Semidirect products and reduction in mechanics*, Trans. Amer. Math. Soc., 281 (1984), pp. 147–177. MR0719663 (84k:58081)
26. M. I. Miller, S. C. Joshi, and G. E. Christensen, *Large deformation fluid diffeomorphisms for landmark and image matching*, in Brain Warping, A. Toga, ed., Academic Press, New York, 1999, pp. 115–131.
27. M. I. Miller, and L. Younes, *Group action, diffeomorphism and matching: A general framework*, Int. J. Comp. Vis, 41 (2001), pp. 61–84.
28. F. Murat and J. Simon, *'Etude de probl'emes d'optimal design*, in Optimization Techniques: Modeling and Optimization in the Service of Man, Lecture Notes in Comput. Sci. 41, Springer-Verlag, New York, 1975, pp. 54–62.
29. B. D. Ripley and A. I. Sutherland, *Finding spiral structures in images of galaxies*, Phil. Trans. Roy. Soc. A, 332 (1990), pp. 477–485.
30. A. Trouv'e, *Infinite Dimensional Group Action and Pattern Recognition*, Tech. report, DMI, Ecole Normale Sup'erieure, Paris, France, 1995. MR1360567 (96j:58019)
31. A. Trouv'e, *Diffeomorphism groups and pattern matching in image analysis*, Int. J. Comp. Vis.,

28 (1998), pp. 213–221.
32. L. Younes, *Computable elastic distances between shapes*, SIAM J. Appl. Math., 58 (1998), pp. 565–586. MR1617630 (99c:68221)

Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.

MR2172454 (2006i:35026) 35F25 (35K55 94A08)

Guichard, Frederic; **Maragos, Petros** (GR-ATHN-SEC);
Morel, Jean-Michel (F-ENSET-AM)

Partial differential equations for morphological operators.

Space, structure and randomness, 369–390, *Lecture Notes in Statist.*, 183, *Springer, New York*, 2005.

This survey paper reviews literature relating certain geometric PDEs to multiscale morphological image processing. Traditionally, the latter was based on modeling images as sets or points in a complete lattice of functions and viewing morphological image transformations as set or lattice operations with corresponding discrete implementations. In the 1990s analogies between these transformations and heat diffusion led to various PDEs that model transformations on multiscale morphological scale-space, that is, a scale indexed family of operators T_t that, in an axiomatic way, represent increased smoothing as t increases. The basic nonlinear morphological operations include flat dilation $f\oplus g(x)=\sup_y(f(y)+g(x-y))$, erosion, $f\ominus g(x)=\inf_y(f(y)-g(y-x))$ and opening ($f\mapsto(f\ominus g)\oplus g$) and closing ($f\mapsto(f\oplus g)\ominus g$). The paper first discusses PDEs that generate these operations, followed by so-called increasing operators, scale space framework and morphological flows and PDEs associated with iterations of increasing operators. Finally, the notion of curve evolution is reviewed. Computational advantages of a PDE-based approach include efficient numerical algorithms that implement PDE morphology on a discrete grid. An example of this is provided.

{For the entire collection see MR2173241 (2006e:62010)}

Reviewed by *Joseph D. Lakey*

MR2169540 (2006d:35222) 35Q53 (35B10 35Q51 37K40 76B25)

Fochesato, Christophe (F-ENSET-AM); **Dias, Frédéric** (F-ENSET-AM);
Grimshaw, Roger (4-LBRO)

Generalized solitary waves and fronts in coupled Korteweg-de Vries systems. (English summary)

Phys. D **210** (2005), *no. 1-2,* 96–117.

Summary: "A variety of problems in nonlinear science can be modelled by a system of two coupled long wave equations. In such systems, a resonance between a solitary wave of one of the two equations and a co-propagating periodic wave of the other equation can occur. The resulting wave is a generalized solitary wave, with non-vanishing oscillatory tails. It is shown that in the case of a 'table-top' solitary wave, which is the solution to an extended Korteweg-de Vries equation with a cubic nonlinearity, generalized solitary waves do not behave like standard sech^2 generalized solitary waves. In particular, it is shown that the oscillations can vanish in the tails or in the central core, but not in both simultaneously. A simplified model is introduced, which allows a better understanding of these stationary long wave solutions and the occurrence of embedded solitons."

MR2165572 (2006c:76029) 76B15 (76S05)

Manam, S. R. (F-ENSET-AM); **Sahoo, T.** (6-IITKH-OEN)

Waves past porous structures in a two-layer fluid. (English summary)

J. Engrg. Math. **52** (2005), *no. 4,* 355–377.

Summary: "Havelock's type of expansion theorems, for an integrable function having a single discontinuity point in the domain where it is defined, are utilized to derive analytical solutions for the radiation or scattering of oblique water waves by a fully extended porous barrier in both the cases of finite and infinite depths of water in two-layer fluid with constant densities. Complete analytical solutions are also obtained for the boundary-value problems dealing with the generation or scattering of axi-symmetric water waves by a system of permeable and impermeable co-axial cylinders. Various results concerning the generation and reflection of the axisymmetric surface or interfacial waves are derived in terms of Bessel functions. The resonance conditions within the trapped region are obtained in various cases. Further, expansions for multipole-line-source oblique-wave potentials are derived for both the cases of finite and infinite depth depending on the existence of the source point in a two-layered fluid."

MR2162865 94A08 (62H35)

Buades, A. (E-BALE); **Coll, B. [Coll, Bartomeu]** (E-BALE); **Morel, J. M.** (F-ENSET-AM)

A review of image denoising algorithms, with a new one. (English summary)

Multiscale Model. Simul. **4** (2005), *no. 2,* 490–530 (*electronic*).

{There will be no review of this item.}

References

1. L. Alvarez, P.-L. Lions, and J.-M. Morel, *Image selective smoothing and edge detection by nonlinear diffusion.* II, SIAM J. Numer. Anal., 29 (1992), pp. 845–866. MR1163360 (93a:35072)
2. J.-F. Aujol, G. Aubert, L. Blanc-F'eraud, and A. Chambolle, *Image decomposition into a bounded variation component and an oscillating component*, J. Math. Imaging Vision, 22 (2005), pp. 71–88. MR2138585 (2005m:68219)
3. S. A. Awate and R. T. Whitaker, *Image denoising with unsupervised, information-theo- retic, adaptive filtering*, in Proceedings of the IEEE International Conference on Computer Vision and Pattern Recognition, 2005, to appear.
4. A. Buades, B. Coll, and J.-M. Morel, *Proc'ed'e de traitement de donn'ees d'image, par r'eduction de bruit d'image, et cam'era int'egrant des moyens de mise en oeuvre du proc'ed'e (Image data process by image noise reduction and camera integrating the means for im- plementing this process)*, French patent application, serial number: 0404837.
5. F. Catt'e, F. Dibos, and G. Koepfler, *A morphological scheme for mean curvature motion and applications to anisotropic diffusion and motion of level sets*, SIAM J. Numer. Anal., 32 (1995), pp. 1875–1909. MR1360464 (96j:65105)
6. A. Chambolle and P. L. Lions, *Image recovery via total variation minimization and related problems*, Numer. Math., 76 (1997), pp. 167–188. MR1440119 (98c:65099)
7. T. F. Chan and H. M. Zhou, *Total variation improved wavelet thresholding in image compression*, in Proceedings of the IEEE International Conference on Image Processing, Vol. 2, Vancouver, BC, Canada, 2000, pp. 391–394.
8. R. R. Coifman and D. Donoho, *Translation-invariant de-noising*, in Wavelets and Statistics, Springer-Verlag, New York, 1995, pp. 125–150.
9. D. Donoho, *De-noising by soft-thresholding*, IEEE Trans. Inform. Theory, 41 (1995), pp. 613–627. MR1331258 (96b:94002)
10. D. Donoho and I. Johnstone, *Ideal spatial adaptation via wavelet shrinkage*, Biometrika, 81 (1994), pp. 425–455. MR1311089 (95m:62076)
11. S. Durand and M. Nikolova, *Restoration of wavelet coefficients by minimizing a specially designed objective function*, in Proceedings of the IEEE Workshop on Variational and Level Set Methods in Computer Vision, 2003.
12. S. Durand and J. Froment, *Reconstruction of wavelet coefficients using total variation min-*

imization, SIAM J. Sci. Comput., 24 (2003), pp. 1754–1767. MR1978159 (2004a:94012)

13. A. Efros and T. Leung, *Texture synthesis by non parametric sampling*, in Proceedings of the IEEE International Conference on Computer Vision, Vol. 2, Corfu, Greece, 1999, pp. 1033–1038.
14. F. Guichard, J. M. Morel, and R. Ryan, *Image Analysis and P.D.E.'s*, preprint.
15. E. Le Pennec and S. Mallat, *Geometrical image compression with bandelets*, in Proceedings of the SPIE 2003, Vol. 5150, Lugano, Switzerland, 2003, pp. 1273–1286.
16. E. Levina, *Statistical Issues in Texture Analysis*, Ph.D. thesis, UC-Berkeley, Berkeley, CA, 2002.
17. M. Lindenbaum, M. Fischer, and A. M. Bruckstein, *On Gabor contribution to image enhancement*, Pattern Recognition, 27 (1994), pp. 1–8.
18. S. Lintner and F. Malgouyres, *Solving a variational image restoration model which involves $L\infty$ constraints*, Inverse Problems, 20 (2004), pp. 815–831. MR2067502 (2005f:94010)
19. F. Malgouyres, *A noise selection approach of image restoration*, in Proceedings of Wavelet Applications in Signal and Image Processing IX, SPIE Proc. Ser. 4478, SPIE, Bellingham, WA, 2001, pp. 34–41.
20. S. Mallat, *A Wavelet Tour of Signal Processing*, Academic Press, San Diego, CA, 1997. MR1614527 (99m:94012)
21. B. Merriman, J. Bence, and S. Osher, *Diffusion generated motion by mean curvature*, in Proceedings of the Geometry Center Workshop, Minneapolis, MN, 1992.
22. Y. Meyer, *Oscillating Patterns in Image Processing and Nonlinear Evolution Equations*, Univ. Lecture Ser. 22, AMS, Providence, RI, 2002. MR1852741 (2002j:43001)
23. 'E. A. Nadaraya, *On non-parametric estimates of density functions and regression curves*, Theory Probab. Appl., 10 (1964), pp. 186–190.
24. E. Ordentlich, G. Seroussi, S. Verd'u, M. Weinberger, and T. Weissman, *A discrete universal denoiser and its application to binary images*, in Proceedings of the IEEE Inter- national Conference on Image Processing, Vol. 1, Barcelona, Spain, 2003, pp. 117–120.
25. S. Osher, M. Burger, D. Goldfarb, J. Xu, and W. Yin, *An iterative regularization method for total variation-based image restoration*, Multiscale Model. Simul., 4 (2005), pp. 460– 489. MR2162864 (2006c:49051)
26. S. Osher, A. Sol'e, and L. Vese, *Image decomposition and restoration using total variation minimization and the H–1 norm*, Multiscale Model. Simul., 1 (2003), pp. 349–370. MR2030155 (2004k:49004)
27. P. Perona and J. Malik, *Scale space and edge detection using anisotropic diffusion*, IEEE Trans. Patt. Anal. Mach. Intell., 12 (1990), pp. 629–639.
28. G. G. Roussas, *Nonparametric regression estimation under mixing conditions*, Stochastic Process. Appl., 36 (1990), pp. 107–116. MR1075604 (91j:62048)
29. L. Rudin and S. Osher, *Total variation based image restoration with free local constraints*, in Proceedings of the IEEE International Conference on Image Processing, Vol. 1, Austin, TX, 1994, pp. 31–35.
30. L. Rudin, S. Osher, and E. Fatemi, *Nonlinear total variation based noise removal algorithms*, Phys. D, 60 (1992), pp. 259–268.

31. C. Shannon and W. Weaver, *The Mathematical Theory of Communication*, University of Illinois Press, Champaign, IL, 1998. MR0032134 (11,258e)
32. A. Sharf, M. Alexa, and D. Cohen-Or, *Context-based surface completion*, in Proceedings of the 31st International Conference on Computer Graphics and Interactive Techniques, Los Angeles, CA, 2004, pp. 878–887.
33. S. M. Smith and J. M. Brady, *SUSAN - a new approach to low level image processing*, International Journal of Computer Vision, 23 (1997), pp. 45–78.
34. J. Starck, E. Cand‘es, and D. Donoho, *The curvelet transform for image denoising*, IEEE Trans. Image Process., 11 (2000), pp. 670–684. MR1929403 (2003j:94013)
35. E. Tadmor, S. Nezzar, and L. Vese, *A multiscale image representation using hierarchical* (*BV,L*2) *decompositions*, Multiscale Model. Simul., 2 (2004), pp. 554–579. MR2113170 (2005h:68163)
36. L. Vese and S. Osher, *Modeling textures with total variation minimization and oscillating patterns in image processing*, J. Sci. Comput., 19 (2003), pp. 553–572. MR2028858 (2004k:49006)
37. G. S. Watson, *Smooth regression analysis*, Sankhy$\bar{a}$ Ser. A, 26 (1964), pp. 359–372. MR0185765 (32 #3226)
38. T. Weissman, E. Ordentlich, G. Seroussi, S. Verdu, and M. Weinberger, *Universal discrete denoising: Known channel*, IEEE Trans. Inform. Theory, 51 (2005), pp. 5–28. MR2234569
39. L. Yaroslavsky, K. Egiazarian, and J. Astola, *Transform domain image restoration meth- ods: Review, comparison and interpretation*, in Nonlinear Image Processing and Pattern Analysis, SPIE, Bellingham, WA, 2001, pp. 155–169.
40. L. P. Yaroslavsky, *Digital Picture Processing. An Introduction*, Springer-Verlag, Berlin, 1985. MR0818222 (88b:94001)
41. L. Yaroslavsky and M. Eden, *Fundamentals of Digital Optics*, Birkh”auser Boston, Boston, MA, 1996.
42. L. Yaroslavsky, *Local adaptive image restoration and enhancement with the use of DFT and DCT in a running window*, in Proceedings of Wavelet Applications in Signal and Image Processing IV, SPIE Proc. Ser. 2825, Denver, CO, 1996, pp. 1–13.

MR2151759 (2006e:49025) 49J45 (49K40 49Q20 54A20 54C10)
Lops, Filomena A. (F-ENSET-AM)
A denoised version of some kinds of set convergence. (English summary)
Adv. Nonlinear Stud. **5** (2005), *no. 3,* 303–335.

In the applications of the classical notions of set convergence—Hausdorff convergence, local Hausdorff convergence and Kuratowski convergence—we encounter the problem that the functionals which have a set as variable and depend on the measure of this set are not lower semicontinuous with respect to these convergences. A typical example is the Mumford-Shah functional and the functional with free discontinuities introduced by De Giorgi and Ambrosio. F. Maddalena and S. Solimini [Arch. Ration. Mech. Anal. **159** (2001), no. 4, 273–294; MR1860049 (2002i:49021)] proved that, for minimizing sequences (K_n) of the Mumford-Shah functional, the only type of obstruction to the lower semicontinuity with respect to the Hausdorff convergence consists of a sequence (K_n^*) of infinitesimal measure contained in (K_n). Maddalena and Solimini [op. cit.] introduced a denoised version of these convergences in such a way as to neglect sets of small measure.

This paper introduces, on a locally compact and σ-compact metric space, denoised versions of Hausdorff convergence, local Hausdorff convergence and Kuratowski convergence, which have the three classical notions as special cases. The author analyzes the connection between the three new notions and proves compactness results. As a first application of these compactness properties, the author introduces a condition, the $\mathcal{H}^s$-almost uniform concentration property on a sequence of sets, which assures the lower semicontinuity of the Hausdorff measure with respect to the denoised Hausdorff convergence of sets. Using the results of Maddalena and Solimini [op. cit.], the author shows that the $\mathcal{H}^s$-almost uniform concentration property is satisfied by any minimizing sequence of Mumford-Shah functionals.

We note that the $\mathcal{H}^s$-almost uniform concentration property is weaker than the one introduced in [G. Dal Maso, J.-M. Morel and S. Solimini, Acta Math. **168** (1992), no. 1-2, 89–151; MR1149865 (92m:49020)] to prove in $\mathbb{R}^2$ the lower semicontinuity of the Hausdorff measure with respect to the Hausdorff metric of sets.

Reviewed by *Fei-Tsen Liang*

MR2149415 (2006a:68161) 68U10 (58D19 68T10)
Trouvé, Alain (F-ENSET-AM); **Younes, Laurent** (1-JHOP-AMS)
Metamorphoses through Lie group action. (English summary)
Found. Comput. Math. **5** (2005), *no. 2,* 173–198.

Summary: "We formally analyze a computational problem which has important applications in image understanding and shape analysis. The problem can be summarized as follows. Starting from a group action on a Riemannian manifold M, we introduce a modification of the metric by partly expressing displacements on M as an effect of the action of some group element. The study of this new structure relates to evolutions on M under the combined effect of the action and of residual displacements, called metamorphoses. This can and has been applied to image processing problems, providing in particular diffeomorphic matching algorithms for pattern recognition."

MR2139148 (2005m:68231) 68U10
Vachier, Corinne (F-ENSET-AM); **Meyer, Fernand** (F-ENSMP3-MR)
The viscous watershed transform. (English summary)
J. Math. Imaging Vision **22** (2005), *no. 2-3,* 251–267.

Summary: "The watershed transform is the basic morphological tool for image segmentation. Watershed lines, also called divide lines, are a topographical concept: a drop of water falling on a topographical surface follows a steepest descent line until it stops when reaching a regional minimum. Falling on a divide line, the same drop of water may glide towards one or the other of the adjacent catchment basins. For segmenting an image, one takes as the topographical surface the modulus of its gradient: the associated watershed lines will follow the contour lines in the initial image. The trajectory of a drop of water is disturbed if the relief is not smooth: it is undefined for instance on plateaus. On the other hand, each regional minimum of the gradient image is the attraction point of a catchment basin. As gradient images generally present many minima, the result is a strong oversegmentation. For these reasons a more robust scheme is used for the construction of the watershed based on flooding: a set of sources is defined, pouring water in such a way that the altitude of the water increases with constant speed. As the flooding proceeds, the boundaries of the lakes propagate in the direction of the steepest descent line of the gradient. The set of points where lakes created by two distinct sources meet are the contours. As the sources are far less numerous than the minima, there is no more oversegmentation, and on the plateaus the flooding also is well defined and propagates from the boundary towards the inside of the plateau. Used in conjunction with markers, the watershed is a powerful, fast and robust segmentation method. Powerful: it has been used with success in a variety of applications. Robust: it is insensitive to

the precise placement or shape of the markers. Fast: efficient algorithms are able to mimic the progression of the flood. In some cases, however, the resulting segmentation will be poor: the contours always belong to the watershed lines of the gradient and these lines are poorly defined when the initial image is blurred or extremely noisy. In such cases, an additional regularization has to take place. Denoising and filtering the image before constructing the gradient is a widely used method. It is, however, not always sufficient. In some cases, one desires smoothing the contour, despite the chaotic fluctuations of the watershed lines. For this two options are possible. The first consists in using a viscous fluid for the flooding: a viscous fluid will not be able to follow all irregularities of the relief and produce lakes with smooth boundaries. Simulating a viscous fluid is, however, computationally intensive. For this reason we propose an alternative solution, in which the topographical surface is modified in such a way that flooding it with a nonviscous fluid will produce the same lakes as flooding the original relief with a viscous fluid. On this new relief, the standard watershed algorithm can be used, which has been optimized for various architectures. Two types of viscous fluids will be presented, yielding two distinct regularization methods. We will illustrate the method on various examples."

MR2138080 (2006b:35087) 35J60 (35B45)
Cerami, G. (I-PBAR); **Devillanova, G.** (F-ENSET-AM); **Solimini, S.** (I-PBAR)
Infinitely many bound states for some nonlinear scalar field equations. (English summary)
Calc. Var. Partial Differential Equations **23** (2005), *no. 2,* 139–168.

The authors study the existence of solutions of the following problem:

(P) $$-\Delta u + a(x)\, u = |u|^{p-2}u \text{ in } \mathbb{R}^N, \quad u \in H^1(\mathbb{R}^N).$$

Here, $N \geq 2$, $p > 2$, and $p < 2N/(N-2)$ when $N \geq 3$. Solutions of (P) can be viewed as solitary waves in nonlinear equations such as the Klein-Gordon equation or the nonlinear Schrödinger equation.

Existence of infinitely many solutions of (P) is proved under the following assumptions on the function $a(x)$: (i) $a \in C^1(\mathbb{R}^N)$; (ii) $\liminf_{|x|\to\infty} a(x) = a_\infty > 0$; (iii) $\frac{\partial a}{\partial \omega}(x)\, e^{\alpha|x|} \to +\infty$ as $|x| \to \infty$ for every $\alpha > 0$; (iv) there exists a constant $c > 1$ such that $|\nabla_\tau a(x)| \leq c\frac{\partial a}{\partial \omega}(x)$ for every $x \in \mathbb{R}^N$ such that $|x| > c$. Here $\omega = x/|x|$ for $x \neq 0$ and $\nabla_\tau a(x)$ denotes the component of the gradient of a at x in the hyperplane orthogonal to ω and containing x. Assumptions (i)–(iv) do not require any symmetry on $a(x)$.

This result is proved by approximating problem (P) by the problems

(P_n) $$-\Delta u + a(x)\, u = |u|^{p-2}u \text{ in } \mathbb{R}^N, \quad u \in H^1_0(B_n),$$

where B_n is a sequence of concentric balls covering $\mathbb{R}^N$. Since each (P_n) admits infinitely many solutions, the conclusion follows by passing to the limit as $n \to \infty$ with the help of a local Pokhozhaev-type inequality and some uniform decay estimates and integral bounds.

Reviewed by *Rolando Magnanini*

References

1. Ambrosetti, A., Badiale, M., Cingolani, S.: Semiclassical states of nonlinear Schrödinger equations. Arch. Rat. Mech. Anal. **140,** 285–300 (1997) MR1486895 (98k:35172)
2. Bahri, A., Lions, P.L.: On the existence of a positive solution of semilinear elliptic equations in unbounded domains. Ann. Ins. H. Poincaré Analyse Nonlineaire **14,** 365–413 (1997) MR1450954 (98k:35047)
3. Bartsch, T., Wang, Z.Q.: Sign changing solutions of nonlinear Schrödinger equations. Top. Meth. Nonlinear Anal. **13,** 191–198 (1999) MR1742220 (2001m:35096)
4. Bartsch, T., Willelm, M.: Infinitely many nonradial solutions of an Euclidean scalar field equation. J. Funct. Anal. **117,** 447–460 (1993) MR1244943 (94k:35093)
5. Benci, V., Cerami, G.: Positive solutions of some nonlinear elliptic problems in exterior domains. Arch. Rat. Mech. Anal. **99,** 283–300 (1987) MR0898712 (88f:35014)
6. Berestycki, H., Lions, P.L.: Nonlinear Scalar field equations. I Existence of a ground state. II Existence of infinitely many solutions. Arch. Rat. Mech. Anal. **82,** 313–346, 347–376 (1983) MR0695535 (84h:35054a)
7. Berger, M.S.: On the existence and structure of stationary states for a nonlinear Klein-Gordon equation. J. Funct. Anal. **9,** 249–261 (1972) MR0299966 (45 #9014)
8. Coffman, C.V.: Uniqueness of the ground state solution for $-\Delta u - u + u^3 = 0$ and a variational characterization of other solutions. Arch. Rat. Mech. Anal. **46,** 81–95 (1972) MR0333489 (48 #11814)
9. Coleman, S., Glazer, V., Martin, A.: Action minima among solutions to a class of Euclidean scalar field equations. Comm. Math. Phys. **58**(2), 211–221 (1978) MR0468913 (57 #8716)
10. Coti Zelati, V., Rabinowitz, P.: Homoclinic type solutions for a semilinear elliptic PDE on $\mathbb{R}^N$. Comm. Pure. Appl. Math. **10,** 1217–1269 (1992) MR1181725 (93k:35087)
11. Devillanova, G., Solimini, S.: Concentrations estimates and multiple solutions to elliptic problems at critical growth. Advances in Differential Equations **7,** 1257–1280 (2002) MR1919704 (2003h:35071)
12. Ding, W.Y., Ni, W.M.: On the existence of positive entire solutions of a semilinear elliptic equation. Arch. Rat. Mech. Anal. **91,** 283–308 (1986) MR0807816 (87b:35056)
13. Floer, A., Weinstein, A.: Nonspreading wave packets for the cubic Schrödinger equation with a bounded potential. J. Funct. Anal. **69,** 397–408 (1986) MR0867665 (88d:35169)
14. Gilbarg, D., Trudinger, N.S.: Elliptic Partial Differential Equations of Second Order. Springer, Berlin Heidelberg New York 1983 MR0737190 (86c:35035)
15. Lions, P.L.: The concentration compactness principle in the calculus of variations Parts I and II. Ann. Ins. H. Poincaré—Analyse Nonlineaire **1,** 109–145, 223–283 (1984) MR0778974 (87e:49035b)
16. Maniscalco, C.: Multiple solutions for semilinear elliptic problems in $\mathbb{R}^N$. Ann. Univ. Ferrara.

37, 95–110 (1991) MR1206996 (94e:35061)
17. Molle, R., Musso, M., Passaseo, D.: Positive solutions for a class of nonlinear elliptic problems in $\mathbb{R}^N$. Proc. Roy. Soc. Edimburgh. A. **130,** 141–166 (2000) MR1742584 (2001e:35058)
18. Nehari, Z.: On a nonlinear differential equation arising in nuclear physics. Proc. Roy. Irish. Acad. **62,** 117–135 (1963) MR0165176 (29 #2465)
19. Oh, Y.G.: On positive multi bump bound states of nonlinear Schrödinger equations under multiple wall potential. Comm. Math. Phys. **131,** 223–253 (1990) MR1065671 (92a:35148)
20. Rabinowitz, P.H.: On a class of nonlinear Schrödinger equations. Z. Angew. Math. Phys. **43,** 270–291 (1992) MR1162728 (93h:35194)
21. Solimini, S.: A note on Compactness-type properties with respect to Lorentz norm of bounded subsets of a Sobolev spaces. Ann. Inst. Henry Poincaré—Analyse Nonlineaire **12,** 319–337 (1995) MR1340267 (96i:46040)
22. Solimini, S.: Morse Index Estimates in min-max theorems. Manuscripta Math. **63,** 421–453 (1989) MR0991264 (90f:58028)
23. Strauss, W.: Existence of solitary waves in higher dimensions. Comm. Math. Phys. **55,** 149–162 (1977) MR0454365 (56 #12616)
24. Struwe, M.: Variational Methods: Applications to nonlinear PDE Hamiltonian Systems. Springer, Berlin Heidelberg New York 1990 MR1078018 (92b:49002)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2134661 (2005k:76085) 76M15 (76B15)
Fochesato, C. (F-ENSET-AM); **Grilli, S. T.** (1-RI-OE); **Guyenne, P.** (3-MMAS)
Note on non-orthogonality of local curvilinear co-ordinates in a three-dimensional boundary element method. (English summary)
Internat. J. Numer. Methods Fluids **48** (2005), *no. 3,* 305–324.

Summary: “We give a more general derivation of the particle velocity and acceleration used in the numerical wave model of S. T. Grilli, P. Guyenne and F. Dias [Internat. J. Numer. Methods Fluids **35** (2001), no. 7, 829–867; Zbl 1039.76043] by expressing these quantities in a local orthogonal co-ordinate system. Computations of solitary waves propagating and breaking over a sloping bottom show that the new formulation gives better results than the former one in the latest stages of overturning. Nevertheless, both formulations are found to be equally suitable for the simulation of non-overturning waves. Results on wave profiles as well as on surface and internal kinematics are presented.”

MR2131291 37N25 (37M10 92C50)
Wesfreid, Eva (F-ENSET-AM); **Billat, Véronique L.** (F-EVRY-STA);
Meyer, Yves (F-ENSET-AM)
Multifractal analysis of heartbeat time series in human races. (English summary)
Appl. Comput. Harmon. Anal. **18** (2005), *no. 3,* 329–335.

{There will be no review of this item.}

References

1. P. Abry, P. Gonçalvès, J. Lévy Véhel, Lois d'échelle, fractales et ondelettes, Hermès Science, ISBN 2–7462-0410-X, 2002.
2. A. Arneodo, F. Argoul, E. Bacry, J. Elezgaray, J.F. Muzzy, Ondelettes multifractales et turbulence, Diderot Éditeur, Paris, 1995.
3. J.B. Bassingthwaighte, L.S. Liebovitch, B.J. West, Fractal Physiology, American Physiology Society, Oxford Univ. Press, 1994, p. 364.
4. V.L. Billat, J. Slawinski, M. Danel, J.P. Koralsztein, Effect of free versus constant pace on performance and oxygen kinetics in running, Med. Sci. Sports Exerc. 33 (2001) 2082–2088.
5. F. Cottin, Y. Papelier, F. Durbin, J.P. Koralsztein, V.L. Billat, Effect of fatigue on spontaneous velocity variations in human middle-distance running: use of short-term Fourier transformation, Eur. J. Appl. Physiol. 87 (2002) 17–27.
6. Y. Fukuba, B.J. Whipp, A metabolic limit on the ability to make up for lost time in endurance events, J. Appl. Physiol. 87 (1999) 853–861.
7. S. Havlin, L.A.N. Amaral, Y. Ashkenazy, A.L. Goldberger, P.Ch. Ivanov, C.K. Peng, H.E. Stanley, Application of statistical physics to heartbeat diagnosis, Physica A 274 (1999) 99–110.
8. P.Ch. Ivanov, M.G. Rosenblum, C.-K. Reng, J. Mietus, S. Havlin, H.E. Stanley, A.L. Goldberger, Scaling behavior of heartbeat intervals obtained by wavelet-based time-series analysis, Nature 383 (1996) 323–327.
9. S. Jaffard, Y. Meyer, R.D. Ryan, Wavelets Tools for Science and Technology, SIAM, ISBN 0–89871-448–6, 2001. MR1827998 (2002g:00007)
10. S. Jaffard, Multifractal formalism for functions Part I: results valid for all functions, SIAM J. Math. Anal. 28 (4) (1997) 944–970; S. Jaffard, Multifractal formalism for functions Part II: self-similar functions, SIAM J. Math. Anal. 28 (4) (1997) 971–998. MR1453315 (99c:28025a)
11. R. Kitney, D. Linkens, A. Selman, A. McDonald, The interaction between heart rate and respiration: Part II—nonlinear analysis based on computer modeling, Automedica 4 (1982) 141–153.
12. http://wave.cmap.polytechnique.fr/soft/LastWave.html.
13. S. Mallat, A Wavelet Tour of Signal Processing, Academic Press, ISBN 0–12-466606-X, 1999.

MR1614527 (99m:94012)
14. M.A. Townsend, Road-racing strategies, Med. Sci. Sports Exerc. 14 (1982) 235–243.
15. F. Xu, E.C. Rhodes, Oxygen uptake kinetics during exercises, Sports Med. 27 (1999) 313–327.
16. Y. Yamamoto, J.O. Fortrat, R.L. Hughson, On the fractal nature of heart rate variability in humans: effect of respiratory sinus arythmia, Am. J. Physiol. 269 (1995) H480–H486.

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2126142 (2006b:35154) 35K55 (49J20 49K20)
Alter, F. (F-ENSET-AM); **Caselles, V.** (E-POFA-T); **Chambolle, A.** (F-PARIS9-A)
Evolution of characteristic functions of convex sets in the plane by the minimizing total variation flow. (English summary)
Interfaces Free Bound. **7** (2005), *no. 1,* 29–53.

In this paper, the authors deal with the study of explicit solutions of a minimizing total variation flow in $\mathbf{R}^2$ given by the equation

$$\frac{\partial u}{\partial t} = \operatorname{div}\left(\frac{Du}{|Du|}\right)$$

in $Q_T = \mathbf{R}^2 \times (0,T)$ with an initial datum which is a characteristic function of a convex set in $\mathbf{R}^2$ or a finite number of convex sets in $\mathbf{R}^2$ which are mutually disjoint.

As an application in image processing, the authors also obtain some explicit solutions for the denoising problem which is given by

$$\min_{u\in \mathrm{BV}(\mathbf{R}^2)} \left\{ \int_{\mathbf{R}^2} |Du| + \frac{1}{2\lambda}\int_{\mathbf{R}^2} (u-f)^2 dx \right\}$$

where $\lambda > 0$, for some data $f \in L^2(\mathbf{R}^2)$.

Some numerical examples of evolutions are given that show agreement with the theoretical results.

Reviewed by *Li Shang Jiang*

MR2123118 (2005m:35035) 35F20 (35B45 76P05 82C40)
Desvillettes, Laurent (F-ENSET-AM); **Mouhot, Clément** (F-ENSLY-PM)
About L^p estimates for the spatially homogeneous Boltzmann equation. (English, French summaries)
Ann. Inst. H. Poincaré Anal. Non Linéaire **22** (2005), *no. 2,* 127–142.

Summary: "For the homogeneous Boltzmann equation with (cutoff or noncutoff) hard potentials, we prove estimates of the propagation of L^p norms with weight $(1+|x|^2)^{q/2}$ $(1<p<\infty, q\in\mathbb{R}_+$ large enough) and of the appearance of such weights. The proof is based on some new functional inequalities for the collision operator, proven by elementary means."

Reviewed by *Hidetoshi Tahara*

References

1. R. Alexandre, L. Desvillettes, C. Villani, B. Wennberg, Entropy dissipation and long range interactions, Arch. Rat. Mech. Anal. 152 (2000) 327–355. MR1765272 (2001c:82061)
2. R. Alexandre, C. Villani, On the Boltzmann equation for long-range interactions, Comm. Pure Appl. Math. 55 (2002) 30–70. MR1857879 (2002f:82026)
3. L. Arkeryd, On the Boltzmann equation, Arch. Rat. Mech. Anal. 45 (1972) 1–34. MR0339665 (49 #4423)
4. L. Arkeryd, Intermolecular forces of infinite range and the Boltzmann equation, Arch. Rat. Mech. Anal. 77 (1981) 11–21. MR0630119 (83k:76057)
5. C. Cercignani, The Boltzmann Equation and its Applications, Springer, 1988. MR1313028 (95i:82082)
6. L. Desvillettes, Some applications of the method of moments for the homogeneous Boltzmann and Kac equations, Arch. Rat. Mech. Anal. 123 (1993) 387–404. MR1233644 (94g:82042)
7. L. Desvillettes, C. Villani, On the spatially homogeneous Landau equation for hard potentials. Part I: Existence, uniqueness and smoothness, Comm. Partial Differential Equations 25 (1/2) (2000) 179–259. MR1737547 (2001c:82065)
8. L. Desvillettes, B. Wennberg, Smoothness of the solution of the spatially homogeneous Boltzmann equation without cutoff, Comm. Partial Differential Equations, submitted for publication. MR2038147 (2004k:82086)
9. M. Escobedo, P. Laurençot, S. Mischler, On a kinetic equation for coalescing particles, Prépublication Inria, 2003. MR2048557 (2005a:82088)
10. T. Goudon, On Boltzmann equations and Fokker-Planck asymptotics: influence of grazing collisions, J. Stat. Phys. 89 (3–4) (1997) 751–776. MR1484062 (98k:82152)
11. T. Gustafsson, L^p-estimates for the nonlinear spatially homogeneous Boltzmann equation, Arch. Rat. Mech. Anal. 92 (1986) 23–57. MR0816620 (87h:82054)
12. T. Gustafsson, Global L^p-properties for the spatially homogeneous Boltzmann equation, Arch. Rat. Mech. Anal. 103 (1988) 1–38. MR0946968 (89h:82019)
13. E. Ikenberry, C. Truesdell, On the pressures and the flux of energy in a gas according to Maxwell's kinetic theory. I, Arch. Rat. Mech. Anal. 5 (1956) 1–54. MR0075725 (17,796c)
14. P.L. Lions, Compactness in Boltzmann's equation via Fourier integral operators and applica-

tions. I, II, III, J. Math. Kyoto Univ. 34 (1994) 539–584. MR1295942 (96f:35134)
15. P.L. Lions, Regularity and compactness for Boltzmann collision operators without angular cut-off, C. R. Acad. Sci. Paris Sér. I 326 (1) (1998) 37–41. MR1649477 (99h:82062)
16. S. Mischler, M. Rodriguez Ricard, Existence globale pour l'équation de Smoluchowski continue non homogène et comportement asymptotique des solutions, C. R. Acad. Sci. Paris Sér. I 336 (2003) 407–412. MR1979355 (2004d:35041)
17. S. Mischler, B. Wennberg, On the spatially homogeneous Boltzmann equation, Ann. Inst. H. Poincaré Anal. Non Linéaire 16 (4) (1999) 467–501. MR1697562 (2000f:35147)
18. C. Mouhot, C. Villani, Regularity theory for the homogeneous Boltzmann with angular cut-off, Arch. Rat. Mech. Anal., submitted for publication.
19. G. Toscani, C. Villani, On the trend to equilibrium for some dissipative systems with slowly increasing a priori bounds, J. Statist. Phys. 98 (2000) 1279–1309. MR1751701 (2001g:82069)
20. G. Toscani, C. Villani, Probability metrics and uniqueness of the solution to the Boltzmann equation for a Maxwell gas, J. Statist. Phys. 94 (1999) 619–637. MR1675367 (2000f:82089)
21. C. Villani, On a new class of weak solutions for the spatially homogeneous Boltzmann and Landau equations, Arch. Rat. Mech. Anal. 143 (1998) 273–307. MR1650006 (99j:82065)
22. C. Villani, Regularity estimates via the entropy dissipation for the spatially homogeneous Boltzmann equation without cut-off, Rev. Mat. Iberoam. 15 (1999) 335–352. MR1715411 (2000f:82090)
23. B. Wennberg, On moments and uniqueness for solutions to the space homogeneous Boltzmann equation, Transport Theory Statist. Phys. 23 (1994) 533–539. MR1264851 (95d:82053)
24. B. Wennberg, Entropy dissipation and moment production for the Boltzmann equation, J. Statist. Phys. 86 (5–6) (1997) 1053–1066. MR1450762 (98k:82167)

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MR2122893 (2005k:03036) 03B42 (68T27)
Walliser, Bernard (F-ENPC-CR); **Zwirn, Denis** (F-POLYP-CR);
Zwirn, Hervé (F-ENSET-AM)
Abductive logics in a belief revision framework. (English summary)
J. Log. Lang. Inf. **14** (2005), *no. 1,* 87–117.

In recent years, a number of authors have explored ways of defining abduction in terms of belief revision. The paper under review surveys three definitions that arise naturally in this context. They are, in effect, variations on a basic pattern, and can be formulated as follows (where K is an arbitrary belief set, $\vdash$ is classical consequence, $*$ is AGM-style belief revision, e is input information (evidence), and h is a candidate hypothesis to which we might abduct).

The 'classical abduction scheme', going back to the earliest days of the literature, requires only $h \vdash e$. From the point of view of belief revision (or equivalently of nonmonotonic inference) it is degenerate, in that only classical consequence is involved, contrasting with the three definitions studied in the paper. These definitions are: $(K * h) \vdash e$ (introduced by Boutilier and Becher in 1995), $h \vdash (K * e)$ (introduced by Cialdea Meyer and Pirri in 1996), and finally $(K * h) \vdash (K * e)$ (studied, along with the others, by Pino Pérez and Uzcátegui in 1999). Thus all four are of the form $((+/-K*)h) \vdash ((+/-K*)e)$.

The paper under review adapts the AGM postulates for belief revision to characterize the relations determined by these three definitions, establishing appropriate representation theorems for them (nontrivial for the last two). The authors also suggest, contra Pino Pérez and Uzcátegui, that the fourth definition (which is also the strongest) provides the most adequate way of understanding abduction in terms of revision.

Reviewed by *David Makinson*

MR2118066 (2005k:80011) 80A32 (76N15 76P05 76T30 76V05)

Desvillettes, L. (F-ENSET-AM); **Monaco, R. [Monaco, Roberto]** (I-TRNP);
Salvarani, F. (I-PAVI)

A kinetic model allowing to obtain the energy law of polytropic gases in the presence of chemical reactions. (English summary)

Eur. J. Mech. B Fluids **24** (2005), *no. 2,* 219–236.

The authors propose a kinetic model of Boltzmann type, describing a mixture of reactive gases, in which a continuous internal energy parameter is present. The model is built in the framework of the so-called Borgnakke-Larsen procedure: nonreactive and reactive collision kernels are written down, focusing on a bimolecular reversible chemical reaction, and conservation laws and the H-theorem are rigorously proved. Finally, the hydrodynamic limit is discussed, and, as a main result, the model enables the authors to recover the Euler equations of a mixture of reactive polytropic perfect gases.

Reviewed by *Maria Groppi*

MR2116276 (2005j:82070) 82C40 (35B40 35F20)
Desvillettes, L. (F-ENSET-AM); **Villani, C.** (F-ENSLY-PM)
On the trend to global equilibrium for spatially inhomogeneous kinetic systems: the Boltzmann equation. (English summary)
Invent. Math. **159** (2005), *no. 2,* 245–316.

In this paper the authors continue the study they initiated in [Comm. Pure Appl. Math. **54** (2001), no. 1, 1–42; MR1787105 (2001h:82079)]. In this instance, the authors apply their method to the Boltzmann equation to obtain convergence of the solutions to the equilibrium distribution faster than $O(t^{-1/\varepsilon})$ for any $\varepsilon > 0$, as $t \to \infty$, under strong smoothness assumptions for the solutions.

The authors use the quantitative versions of Boltzmann's H theorem from [C. Villani, Comm. Math. Phys. **234** (2003), no. 3, 455–490; MR1964379 (2004b:82048)]—which gives an explicit bound for the entropy production functional—and establish the convergence of $H(f)$ with explicit rates as above. The result then follows from the Csiszár-Kullback-Pinsker inequality. The technique also depends on previous work developed by the authors in [op. cit., 2001] and [ESAIM Control Optim. Calc. Var. **8** (2002), 603–619 (electronic); MR1932965 (2004i:82053)], estimates on some systems of second-order differential inequalities and a Korn-like estimate used to prove what the authors call the "instability of the hydrodynamical regime", the fact that the solution does not stay close to a local Maxwellian for very long.

This paper is a major contribution to the field of kinetic equations. Even taking into account the strong smoothness assumptions, this is the first result providing quantitative estimates for the entropy production in this context. It is a beautiful result presented in a very clear way.

Reviewed by *Manuel Portilheiro*

References

1. Alexandre, R., Villani, C.: On the Boltzmann equation for long-range interactions. Commun. Pure Appl. Math. **55,** 30–70 (2002) MR1857879 (2002f:82026)
2. Alexandre, R., Villani, C.: On the Landau approximation in plasma physics. Ann. Inst. Henri Poincaré, Anal. Non Linéaire **21,** 61–95 (2004) MR2037247 (2005f:82126)
3. Arkeryd, L.: Stability in L^1 for the spatially homogeneous Boltzmann equation. Arch. Ration. Mech. Anal. **103,** 151–167 (1988) MR0946973 (89h:82015)
4. Arkeryd, L., Nouri, A.: Boltzmann asymptotics with diffuse reflection boundary conditions. Monatsh. Math. **123,** 285–298 (1997) MR1448572 (98h:35225)
5. Baranger, C., Mouhot, C.: Explicit spectral gap estimates for the Boltzmann and Landau operators. To appear in Rev. Mat. Iberoam.
6. Bardos, C., Golse, F., Levermore, D.: Fluid dynamical limits of kinetic equations, II: Convergence proofs for the Boltzmann equation. Commun. Pure Appl. Math. **46,** 667–753 (1993) MR1213991 (94g:82039)

7. Brush, S.: Kinetic Theory, Vol. 2: Irreversible Processes. Oxford: Pergamon Press 1966
8. Cáceres, M.-J., Carrillo, J.-A., Goudon, T.: Equilibration rate for the linear inhomogeneous relaxation-time Boltzmann equation for charged particles. Commun. Partial Differ. Equations **28,** 969–989 (2003) MR1986057 (2004g:82111)
9. Caflisch, R.: The Boltzmann equation with a soft potential. I. linear, spatially-homogeneous. II. nonlinear, spatially-periodic. Commun. Math. Phys. **74,** 71–95, 97–109 (1980) MR0575897 (82j:82040a)
10. Carleman, T.: Sur la théorie de l'equation intégrodifférentielle de Boltzmann. Acta Math. **60,** 369–424 (1932)
11. Carlen, E., Carvalho, M.: Strict entropy production bounds and stability of the rate of convergence to equilibrium for the Boltzmann equation. J. Stat. Phys. **67,** 575–608 (1992) MR1171145 (93h:82049)
12. Carlen, E., Carvalho, M.: Entropy production estimates for Boltzmann equations with physically realistic collision kernels. J. Stat. Phys. **74,** 743–782 (1994) MR1263387 (95g:82073)
13. Carlen, E., Carvalho, M., Loss, M.: Determination of the spectral gap for Kac's master equation and related stochastic evolutions. Acta Math. **191,** 1–54 (2003) MR2020418 (2004j:60186)
14. Cercignani, C.: Ludwig Boltzmann, the man who trusted atoms. New York: Oxford University Press 1998 MR1700775 (2000f:01023)
15. Cercignani, C.: Rarefied gas dynamics. From basic concepts to actual calculations. Cambridge: Cambridge University Press 2000 MR1744523 (2001f:76065)
16. Degond, P., Pareschi, L., Russo, G. (eds.): Modeling and computational methods for kinetic equations. Birkhäuser 2003 MR2069221 (2005a:82001)
17. Desvillettes, L.: Entropy dissipation rate and convergence in kinetic equations. Commun. Math. Phys. **123,** 687–702 (1989) MR1006301 (90g:82050)
18. Desvillettes, L.: Convergence to equilibrium in large time for Boltzmann and BGK equations. Arch. Ration. Mech. Anal. **110,** 73–91 (1990) MR1031086 (91d:35210)
19. Desvillettes, L.: Convergence to equilibrium in various situations for the solution of the Boltzmann equation. In: Nonlinear kinetic theory and mathematical aspects of hyperbolic systems (Rapallo, 1992), pp. 101–114. River Edge, NJ: World Sci. Publishing 1992 MR1240098 (94j:82064)
20. Desvillettes, L., Villani, C.: On the spatially homogeneous Landau equation for hard potentials. II. H-theorem and applications. Commun. Partial Differ. Equations **25,** 261–298 (2000) MR1737548 (2001c:82066)
21. Desvillettes, L., Villani, C.: On the trend to global equilibrium in spatially inhomogeneous entropy-dissipating systems: the linear Fokker-Planck equation. Commun. Pure Appl. Math. **54,** 1–42 (2001) MR1787105 (2001h:82079)
22. Desvillettes, L., Villani, C.: On a variant of Korn's inequality arising in statistical mechanics. ESAIM, Control Optim. Calc. Var. **8,** 603–619 (2002) (electronic). A tribute to J.L. Lions MR1932965 (2004i:82053)
23. Diaconis, P., Saloff-Coste, L.: Bounds for Kac's master equation. Commun. Math. Phys. **209,** 729–755 (2000) MR1743614 (2002e:60107)
24. DiPerna, R., Lions, P.-L.: On the Cauchy problem for the Boltzmann equation: Global existence

and weak stability. Ann. Math. (2) **130,** 312–366 (1989) MR1014927 (90k:82045)
25. Fellner, K., Neumann, L., Schmeiser, C.: Convergence to global equilibrium for spatially inhomogeneous kinetic models of non-micro-reversible processes. Monatsh. Math. **141,** 289–299 (2004) MR2053654 (2005f:82116)
26. Filbet, F., Russo, G.: High order numerical methods for the space non-homogeneous Boltzmann equation. J. Comput. Phys. **186,** 457–480 (2003) MR1973198 (2004c:82116)
27. Filbet, F.: Quelques résultats numériques sur l'équation de Boltzmann non homogène. Preprint 2004
28. Gallay, T., Wayne, C.: Invariant manifolds and the long-time asymptotics of the Navier-Stokes and vorticity equations on $\mathbb{R}^2$. Arch. Ration. Mech. Anal. **163,** 209–258 (2002) MR1912106 (2003c:37123)
29. Golse, F., Saint-Raymond, L.: The Navier-Stokes limit of the Boltzmann equation: Convergence proof. Invent. Math. **155,** 81–161 (2004) MR2025302 (2005f:76003)
30. Grad, H.: On Boltzmann's H-theorem. J. Soc. Indust. Appl. Math. **13,** 259–277 (1965) MR0180278 (31 #4513)
31. Guo, Y.: The Landau equation in a periodic box. Commun. Math. Phys. **231,** 391–434 (2002) MR1946444 (2004c:82121)
32. Guo, Y.: Classical solutions to the Boltzmann equation for molecules with an angular cutoff. Arch. Ration. Mech. Anal. **169,** 305–353 (2003) MR2013332 (2004i:82054)
33. Guo, Y.: The Vlasov-Maxwell-Boltzmann system near Maxwellians. Invent. Math. **153,** 593–630 (2003) MR2000470 (2004m:82123)
34. Guo, Y.: Private communication (December 2003)
35. Gustafsson, T.: L^p-estimates for the nonlinear spatially homogeneous Boltzmann equation. Arch. Ration. Mech. Anal. **92,** 23–57 (1986) MR0816620 (87h:82054)
36. Helffer, B., Nier, F.: Hypoellipticity and spectral theory for Fokker-Planck operators and Witten Laplacians. Preprint 03–25, Université de Rennes (2003). Available at http://name.math.univ-rennes1.fr/francis.nier/recherche/liste.html cf. MR 2005c:47070
37. Hérau, F., Nier, F.: Isotropic hypoellipticity and trend to the equilibrium for the Fokker-Planck equation with high degree potential. Arch. Ration. Mech. Anal. **171,** 151–218 (2004) MR2034753 (2005f:82085)
38. Janvresse, E.: Spectral gap for Kac's model of Boltzmann equation. Ann. Probab. **29,** 288–304 (2001) MR1825150 (2002c:82068)
39. Kac, M.: Foundations of kinetic theory. In: Proceedings of the Third Berkeley Symposium on Mathematical Statistics and Probability, 1954–1955, vol. III, pp. 171–197. Berkeley, Los Angeles: University of California Press 1956 MR0084985 (18,960i)
40. Lions, P.-L., Masmoudi, N.: From the Boltzmann equations to the equations of incompressible fluid mechanics. I, II. Arch. Ration. Mech. Anal. **158,** 173–193, 195–211 (2001) MR1842343 (2002m:76085)
41. Lu, X.: Spatial decay solutions of the Boltzmann equation: converse properties of long time limiting behavior. SIAM J. Math. Anal. **30,** 1151–1174 (1999) MR1709791 (2000k:35274)
42. McKean, H. J.: Speed of approach to equilibrium for Kac's caricature of a Maxwellian gas. Arch. Ration. Mech. Anal. **21,** 343–367 (1966) MR0214112 (35 #4963)

43. Mouhot, C.: Quantitative lower bounds for the full Boltzmann equation. Preprint 2003 cf. MR 2004f:68039
44. Mouhot, C., Villani, C.: Regularity theory for the spatially homogeneous Boltzmann equation with cutoff. Arch. Ration. Mech. Anal., in press
45. Pitteri, M.: On the asymptotic behaviour of Boltzmann's H function in the kinetic theory of gases. Rend. Sc. Fis. Mat. e Nat. **67,** 248–251 (1979) MR0622800 (83b:82052)
46. Poincaré, H.: Le mécanisme et l'expérience. Revue de Métaphysique et de Morale **I,** 534–537 (1893)
47. Pulvirenti, A., Wennberg, B.: A Maxwellian lower bound for solutions to the Boltzmann equation. Commun. Math. Phys. **183,** 145–160 (1997) MR1461954 (99f:82056)
48. Shizuta, Y., Asano, K.: Global solutions of the Boltzmann equation in a bounded convex domain. Proc. Japan Acad., Ser. A **53,** 3–5 (1977) MR0466988 (57 #6861)
49. Spohn, H.: Large scale dynamics of interacting particles. Texts and Monographs in Physics. Berlin: Springer 1991
50. Sznitman, A.-S.: Topics in propagation of chaos. In: École d'Été de Probabilités de Saint-Flour XIX–1989, pp. 165–251. Berlin: Springer 1991 MR1108185 (93b:60179)
51. Toscani, G.: H-theorem and asymptotic trend to equilibrium of the solution for a rarefied gas in the vacuum. Arch. Ration. Mech. Anal. **100,** 1–12 (1987) MR0906131 (89b:82053)
52. Toscani, G., Villani, C.: Sharp entropy dissipation bounds and explicit rate of trend to equilibrium for the spatially homogeneous Boltzmann equation. Commun. Math. Phys. **203,** 667–706 (1999) MR1700142 (2000e:82039)
53. Toscani, G., Villani, C.: On the trend to equilibrium for some dissipative systems with slowly increasing a priori bounds. J. Stat. Phys. **98,** 1279–1309 (2000) MR1751701 (2001g:82069)
54. Truesdell, C., Muncaster, R.: Fundamentals of Maxwell's kinetic theory of a simple monoatomic gas. New York: Academic Press 1980 MR0554086 (81f:80001)
55. Ukai, S.: On the existence of global solutions of mixed problem for non-linear Boltzmann equation. Proc. Japan Acad. **50,** 179–184 (1974) MR0363332 (50 #15770)
56. Villani, C.: Fisher information bounds for Boltzmann's collision operator. J. Math. Pures Appl. **77,** 821–837 (1998) MR1646804 (2000j:82033)
57. Villani, C.: Limites hydrodynamiques de l'équation de Boltzmann (d'après C. Bardos, F. Golse, C. D. Levermore, P.-L. Lions, N. Masmoudi, L. Saint-Raymond). Astérisque **282** (2002), Exp. No. 893, ix, 365–405. Séminaire Bourbaki, Vol. 2000/2001 MR1975186 (2005f:82108)
58. Villani, C.: A review of mathematical topics in collisional kinetic theory. In: Handbook of mathematical fluid dynamics, Vol. I, pp. 71–305. Amsterdam: North-Holland 2002 MR1942465 (2003k:82087)
59. Villani, C.: On the Boltzmann equation with singular kernel. Unpublished notes
60. Villani, C.: Cercignani's conjecture is sometimes true and always almost true. Commun. Math. Phys. **234,** 455–490 (2003) MR1964379 (2004b:82048)
61. Villani, C.: Topics in Optimal Transportation, vol. 58 of Graduate Series in Mathematics. Providence: American Mathematical Society 2003 MR1964483 (2004e:90003)
62. Wennberg, B.: Stability and exponential convergence in L^p for the spatially homogeneous Boltzmann equation. Nonlinear Anal. **20,** 935–964 (1993) MR1215007 (94h:76076)

63. Wennberg, B.: Stability and exponential convergence for the Boltzmann equation. Arch. Ration. Mech. Anal. **130,** 103–144 (1995) MR1338453 (96i:82137)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2102914 (2005m:76121) 76M12 (65M06)

Ghidaglia, Jean-Michel (F-ENSET-AM); **Pascal, Frédéric** (F-ENSET-AM)

The normal flux method at the boundary for multidimensional finite volume approximations in CFD. (English summary)

Eur. J. Mech. B Fluids **24** (2005), *no. 1,* 1–17.

Summary: "This paper presents a general method for imposing boundary conditions in the context of hyperbolic systems of conservation laws. This method is particularly well suited for approximations in the framework of finite volume methods in the sense that it computes directly the normal flux at the boundary. We generalize our approach to nonconservative hyperbolic systems and discuss both the characteristic and the noncharacteristic cases. We present several applications to models occurring in computational fluid mechanics like the Euler equations for compressible inviscid fluids with real equation of state, shallow water equations, magnetohydrodynamics equations and two fluid models."