MR2294796 46E35 (47H30)
Bourdaud, Gérard (F-PARIS7-ANF); **Meyer, Yves** (F-ENSET-AM)

Le calcul fonctionnel sous-linéaire dans les espaces de Besov homogènes. (French. French summary) [Sublinear functional calculus in homogeneous Besov spaces]

Rev. Mat. Iberoamericana **22** (2006), *no. 2,* 725–746, *loose erratum*.

{A review for this item is in process.}

References

1. Besov, O. V. : Investigation of a class of function spaces in connection with imbedding and extension theorems. (Russian) Trudy. Mat. Inst. Steklov **60** (1961), 42–81. MR0133675 (24 #A3501)
2. Bony, J. M. : Calcul symbolique et propagation des singularit'es pour les 'equations aux d'eriv'ees partielles non lin'eaires. Ann. Sci. 'Ecole Norm. Sup. (4) **14** (1981), 209–246. MR0631751 (84h:35177)
3. Bourdaud, G. : R'ealisations des espaces de Besov homog'enes. Ark. Mat. **26** (1988), 41–54. MR0948279 (90d:46046)
4. Bourdaud, G. : Fonctions qui op'erent sur les espaces de Besov et de Triebel. Ann. Inst. H. Poincar'e Anal. Non Lin'eaire **10** (1993), 413–422. MR1246460 (94m:46055)
5. Bourdaud, G. : Ce qu'il faut savoir sur les espaces de Besov. Pr'epub. Univ. Paris 7 - URA 212, **53** (1993).
6. Bourdaud, G. : The functional calculus in Sobolev spaces. In Function spaces, differential operators and non linear analysis (Friedrichroda, 1992), 127–142. Teubner-Texte Math. **133**. Teubner, Stuttgart, 1993. MR1242580 (94e:46055)
7. Bourdaud, G. : Calcul fonctionnel dans certains espaces de Lizorkin- Triebel. Arch. Math. (Basel) **64** (1995), 42–47. MR1305659 (96a:46064)
8. Bourdaud, G. : Ondelettes et espaces de Besov. Rev. Mat. Iberoameri- cana **11** (1995), no. 3, 477–512. MR1363202 (97b:42048)
9. Bourdaud, G. : Superposition operators in Zygmund and BMO spaces. In Function spaces, differential operators and non linear analysis (Teistungen, 2001), 59–74. Birkh"auser, Basel, 2003. MR1984163 (2004i:47123)
10. Bourdaud, G. and Kateb, M. E. D. : Calcul fonctionnel dans l'espace de Sobolev fractionnaire. Math. Z. **210** (1992), 607–613. MR1175725 (93i:46058)
11. Bourdaud, G. and Kateb, M. E. D. : Fonctions qui op'erent sur les es- paces de Besov. Math. Ann. **303** (1995), 653–675. MR1359954 (96k:47118)
12. Bourdaud, G. and Lanza de Cristoforis, M. : Functional calculus in H"older-Zygmund spaces. Trans. Amer. Math. Soc. **354** (2002), 4109–4129. MR1926867 (2004d:47112)
13. Bourdaud, G., Lanza de Cristoforis, M. and Sickel, W. : Super- position operators and functions of bounded p-variation. Rev. Mat. Iberoa- mericana, **22** (2006), no. 2, 455–487. MR2294787

14. Bourdaud, G. and Meyer, Y. : Fonctions qui op'erent sur les espaces de Sobolev. J. Funct. Anal. **97** (1991), 351–360. MR1111186 (92e:46062)
15. Chemin, J.-Y. : Fluides parfaits incompressibles. Ast'erisque **230** (1995). MR1340046 (97d:76007)
16. DeVore, R. A. and Lorentz, G. G. : Constructive approximation. Fun- damental Principles of Mathematical Sciences **303**. Springer, Berlin, 1993. MR1261635 (95f:41001)
17. Franke, J. : On the space *F spq* of Triebel-Lizorkin type : pointwise multi- pliers and spaces on domains. Math. Nachr. **125** (1986), 29–68. MR0847350 (88e:46029)
18. Lemari'e-Rieusset, P. G. : Recent developments in the Navier-Stokes pro- blem. Chapman Hall/CRC Research Notes in Mathematics **431**. Chapman Hall/CRC, Boca Raton, FL, 2002. MR1938147 (2004e:35178)
19. Meyer, Y. : Oscillating patterns in image processing and nonlinear evolu- tion equations. The fifteenth Dean Jacqueline B. Lewis memorial lectures. University Lecture Series, **22**, Amer. Math. Soc., Providence, RI, 2001. MR1852741 (2002j:43001)
20. Oswald, P. : On the boundedness of the mapping $f \longrightarrow$ —f— in Besov spaces. Comment. Math. Univ. Carolin. **33** (1992), 57–66. MR1173747 (93c:46052)
21. Peetre, J. : Interpolation of Lipschitz operators and metric spaces. Ma- thematica (Cluj) **12** (1970), 325–334. MR0482280 (58 #2354)
22. Peetre, J. : New thoughts on Besov spaces. Duke Univ. Math. Series I. Mathematics Department, Duke University, Durham, N.C., 1976. MR0461123 (57 #1108)
23. Runst, T. and Sickel, W. : Sobolev spaces of fractional order, Nemytskij operators, and nonlinear partial differential equations. De Gruyter Series in Nonlinear Analysis and Applications **3**. Walter de Gruyter, Berlin, 1996. MR1419319 (98a:47071)
24. Savar'e, G. : On the regularity of the positive part of functions. Nonlinear Anal. **27** (1996), no. 9, 1055–1074. MR1406279 (97i:46066)
25. Savar'e, G. and Tomarelli, F. : Superposition and chain rule for boun- ded Hessian functions. Adv. Math. **140** (1998), 237–281. MR1658593 (99k:35005)
26. Triebel, H. : Interpolation theory, function spaces, differential operators. North-Holland Mathematical Library **18**. North-Holland, Amsterdam-New York, 1978. MR0503903 (80i:46032b)
27. Triebel, H. : Theory of function spaces II. Monographs in Mathema- tics **84**. Birkh"auser, Basel, 1992. MR1163193 (93f:46029)

MR2287348 94A08 (92Cxx)

Masnou, Simon (F-PARIS6-N); **Morel, Jean-Michel** (F-ENSET-AM)

On a variational theory of image amodal completion. (English summary)

Rend. Sem. Mat. Univ. Padova **116** (2006), 211–252.

{A review for this item is in process.}

MR2276075 93B05 (35J10 35Q40 81Q05)

Beauchard, Karine (F-ENSET-AM)

Controllability of Shrödinger equations. (English summary) [Controllability of Schrödinger equations]

Seminaire: *Equations aux Dérivées Partielles*. 2005–2006, *Exp*. *No*. *IX*, 20 *pp*., *Sémin*. *Équ*. *Dériv*. *Partielles*, *École Polytech*., *Palaiseau*, 2006.

{A review for this item is in process.}
{For the entire collection see MR2276066 (2007f:35007)}

MR2277067 (2007k:37081) 37J45 (37K10 37M99 53D12)

Chardard, F. (F-ENSET-AM); **Dias, F. [Dias, Frédéric]** (F-ENSET-AM); **Bridges, T. J.** (4-SUR)

Fast computation of the Maslov index for hyperbolic linear systems with periodic coefficients. (English summary)

J. Phys. A **39** (2006), *no. 47,* 14545–14557.

The Maslov index is a powerful tool to study periodic solutions of Hamiltonian systems. But the concrete computation of the index of periodic solutions of related nonlinear problems is usually very difficult. Among all the linear Hamiltonian systems, hyperbolic ones are always nondegenerate and are among the simpler systems. This paper is devoted to establishing a numerical scheme to compute the Maslov index for hyperbolic linear Hamiltonian systems with periodic coefficients. The main idea is that upon considering the exterior algebra of the ambient vector space, the Lagrangian subspace representing the unstable subspace reduces to a line, and then it forms a closed loop when the exterior algebra is projectified. This result is applied to the computation of the Maslov index for the spectral problem associated with periodic solutions of the fifth-order Korteweg-de Vries equation.

Reviewed by *Yiming Long*

MR2274202 (2007i:53073) 53C44 (37K25)

Glaunès, Joan (F-PARIS13-AG); **Trouvé, Alain** (F-ENSET-AM);
Younes, Laurent (1-JHOP-CIS)

Modeling planar shape variation via Hamiltonian flows of curves. (English summary)

Statistics and analysis of shapes, 335–361, *Model. Simul. Sci. Eng. Technol.*, *Birkhäuser Boston, Boston, MA*, 2006.

The application of the theory of deformable templates to the study of the action of a group of diffeomorphisms on deformable objects provides a framework for computing dense one-to-one matchings on d-dimensional domains.

In the paper under review the geodesic equations governing the time evolution of an optimal matching are derived in the case of the action on 2D curves with various driving matching terms. A Hamiltonian formulation is provided in which the initial momentum is represented by an L^2 vector field on the boundary of the template.

{For the entire collection see MR2274188 (2007f:62008)}

Reviewed by *Riccardo De Arcangelis*

MR2274193 68T45 (51M99 62H35 68T10 68U10)

Musé, Pablo (F-ENSET-AM); **Sur, Frédéric** (F-ENSET-AM); **Cao, Frédéric** (F-RENNB-II); **Gousseau, Yann** (F-ENST-TS); **Morel, Jean-Michel** (F-ENSET-AM)

Shape recognition based on an a contrario methodology. (English summary)

Statistics and analysis of shapes, 107–136, *Model. Simul. Sci. Eng. Technol.*, *Birkhäuser Boston, Boston, MA*, 2006.

{This item will not be reviewed individually. For details of the collection in which this item appears see MR2274188 .}

{For the entire collection see MR2274188}

MR2269565 (2007m:82084) 82C70 (35B25 35F20 82C40)

Bisi, M. [Bisi, Marzia] (I-PARM); **Desvillettes, L.** (F-ENSET-AM)

From reactive Boltzmann equations to reaction-diffusion systems. (English summary)

J. Stat. Phys. **125** (2006), *no. 1,* 249–280.

This paper is concerned with the reactive Boltzmann equation, an extension of the classical Boltzmann equation, for a mixture of different species of molecules with chemical reactions. The authors consider the case when the collisions involving one molecule (whose distribution is a Maxwellian) are predominant. Formally, under suitable scalings, it is shown that the solutions of the Boltzmann equation converge to the solutions of the reaction-diffusion system. Also, for a simplified system a rigorous proof is given.

Reviewed by *Fucai Li*

MR2270649 (2007h:76017) 76B15 (35Q35 37N10 76Mxx)
Dias, Frédéric (F-ENSET-AM); **Bridges, Thomas J.** (4-SUR-MS)
The numerical computation of freely propagating time-dependent irrotational water waves. (English summary)
Fluid Dynam. Res. **38** (2006), *no. 12,* 803–830.

Introduction: "Water waves have long been studied because of their practical importance and because they offer an ideal setting for a variety of phenomena of nonlinear wave motion.

"Compared to the analytical study of the water-wave problem, which was initiated at the beginning of the nineteenth century, the numerical study of water waves is relatively recent for obvious reasons. The first numerical computations were performed in the 1970s, with a few exceptions in the 1960s. Since then, the steady progress in the power of computers combined with the development of more and more efficient numerical methods has allowed researchers to tackle problems closer and closer to real-life situations.

"One can distinguish four major classes of numerical computations dealing with water waves: (i) computations of progressive waves that propagate without changing form (time dependence can be removed from the equations by working in a frame of reference moving with the wave), (ii) computations of standing waves (these waves are periodic in time and in space, but time dependence cannot be removed from the equations), (iii) computations aimed at a statistical description of water waves (such computations require integration over a long time), and (iv) numerical wave tanks that are designed to mimic laboratory wave tanks or even the ocean in the sense that arbitrary wave motion can be described. For each case, one can make a further distinction between two-dimensional waves and three-dimensional waves.

"The emphasis of the review is on the numerical computation of freely propagating time-dependent waves. This is a rich subject and bias is unavoidable. The mathematical model that is used in the present review is that of inviscid irrotational flow. The governing equations are the incompressible, irrotational Euler equations in the presence of a free surface. Among the numerical methods and equations based on this mathematical model, some are closer to the model than others. In particular, approximate equations such as the Korteweg-de Vries equation, the Boussinesq equation, the nonlinear Schrödinger equation and their variants represent a further approximation. Although they have been shown to provide very good results in a variety of applications, they are not considered here. Moreover, by restricting ourselves to freely propagating waves, we do not discuss the interactions of water waves with structures, a topic of great industrial interest. For earlier reviews on water waves or on the numerical simulation of free-surface flows, one can refer to [L. W. Schwartz and J. D. Fenton, in *Annual review of fluid mechanics, Vol. 14*, 39–60, Annual Reviews, Palo Alto, Calif., 1982; MR0642535 (83b:76016); W.-T. Tsai and D. K. P. Yue, in *Annual review of fluid mechanics, Vol. 28*, 249–278, Annual Reviews, Palo Alto, CA, 1996; MR1371167 (96i:76086); F. Dias and C. Kharif, in *Annual review of fluid mechanics, Vol. 31*, 301–346, Annual Reviews, Palo Alto, CA, 1999; MR1670945 (99k:76024); R. Scardovelli and S. Zaleski, in *Annual review of fluid mechanics, Vol. 31*, 567–603, Annual Reviews, Palo Alto, CA, 1999; MR1670950 (99m:76002); D. H. Peregrine, in *Annual review of fluid mechanics, Vol. 35*, 23–43, Annual Reviews, Palo Alto, CA, 2003; MR1967007 (2004b:76027)].”

MR2267194 (2007g:76023) 76B15 (35Q35 35R35)

Dutykh, Denys (F-ENSET-AM); **Dias, Frédéric** (F-ENSET-AM); **Kervella, Youen** (F-BRET)

Linear theory of wave generation by a moving bottom. (English, French summaries)

C. R. Math. Acad. Sci. Paris **343** (2006), *no. 7,* 499–504.

Summary: “The computation of long wave propagation through the ocean obviously depends on the initial conditions. When the waves are generated by a moving bottom, a traditional approach consists in translating the ‘frozen’ sea bed deformation to the free surface and propagating it. The present study shows the differences between the classical approach (passive generation) and the active generation where the bottom motion is included. The analytical solutions presented here exhibit some of the drawbacks of passive generation. The linearized solutions seem to be sufficient to consider the generation of water waves by a moving bottom.”

References

1. D. Dutykh, F. Dias, Water waves generated by a moving bottom, in: A. Kundu (Ed.), Tsunami and Nonlinear Waves, Geosciences, Springer-Verlag, 2006, in press.
2. L.N.G. Filon, On a quadrature formula for trigonometric integrals, Proc. Royal Soc. Edinburgh 49 (1928) 38–47.
3. J.L. Hammack, A note on tsunamis: their generation and propagation in an ocean of uniform depth, J. Fluid Mech. 60 (1973) 769–799.
4. Y. Okada, Internal deformation due to shear and tensile faults in a half-space, Bull. Seism. Soc. Am. 82 (1992) 1018–1040.

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2257384 94A08 (35F20 65D18 68U10)
Buades, Antoni (E-BALE); **Coll, Bartomeu** (E-BALE); **Morel, Jean-Michel** (F-ENSET-AM)
Neighborhood filters and PDE's. (English summary)
Numer. Math. **105** (2006), *no. 1,* 1–34.

{A review for this item is in process.}

References

1. Amann, H.: A new approach to quasilinear parabolic problems. In: International Conference on Differential Equations, 2005
2. Andreu, F., Ballester, C., Caselles, V., Mazon, J. M.: Minimizing Total Variation Flow. Differ. Integral Equations **14**(3), 321–360 (2001) MR1799898 (2002e:35109)
3. Di Zenzo, S.: A note on the gradient of a multi-image. Comput. Vis. Graph. Image Process. **33,** 116–125 (1986)
4. Esedoglu, S.: An analysis of the Perona–Malik scheme. Commun. Pure Appl. Math. **54,** 1442–1487 (2001) MR1852979 (2003i:94005)
5. Guichard, F., Morel, J. M.: A note on two classical enhancement filters and their associated PDE's. Int. J. Comput. Vis. **52**(2–3), 153–160 (2003)
6. Harten, A., Enquist, B., Osher, S., Chakravarthy, S.: Uniformly high order accurate essentially non-oscillatory schemes III. J. Comput. Phys. **71,** 231–303 (1987) MR0897244 (90a:65199)
7. Kichenassamy, S.: The Perona–Malik paradox. SIAM J. Appl. Math. **57**(2), 1328–1342 (1997) MR1470926 (98i:35088)
8. Kimia, B.B., Tannenbaum, A., Zucker, S.W.: On the evolution of curves via a function of curvature I the classical case. J. Math. Anal. Appl. **163**(2), 438–458 (1992) MR1145840 (93a:58037)
9. Kimmel, R., Malladi, R., Sochen, N.: Images as embedded maps and minimal surfaces: movies, color, texture, and volumetric medical images. Int. J. Comput. Vis. **39**(2), 111–129 (2000)
10. Kindermann, S., Osher, S., Jones, P.: Deblurring and denoising of images by nonlocal functionals. UCLA Computational and Applied Mathematics Reports 04–75, 2004
11. Kornprobst, P.: Contributions la Restauration d'Images et l'Analyse de Sequences: Approches Variationnelles et Solutions de Viscosite. PhD thesis, Universite de Nice-Sophia Antipolis, 1998
12. Kramer, H.P., Bruckner, J.B.: Iterations of a non-linear transformation for enhancement of digital images. Pattern Recogn. **1–2,** 53–58, (1975) MR0386379 (52 #7233)
13. Lee, J.S.: Digital image smoothing and the sigma filter. Comput. Vis. Graph. Image Process. **24,** 255–269 (1983)
14. Masnou, S.: Filtrage et désocclusion d'images par méthodes d'ensembles de niveau. PhD Dissertation, Université Paris-IX Dauphine, 1998
15. Osher, S., Rudin, L.: Feature oriented image enchancement using shock filters. SIAM J Numer. Anal. **27,** 919–940 (1990)
16. Perona, P., Malik, J.: Scale space and edge detection using anisotropic diffusion. IEEE Trans. Pattern Anal. Mach. Intell. **12,** 629–639 (1990)
17. Rudin, L., Osher, S., Fatemi, E.: Nonlinear total variation based noise removal algorithms.

Physica D **60,** 259–268 (1992)
18. Saint-Marc, P., Chen, J.S., Medioni, G.: Adaptive smoothing: a general tool for early vision. IEEE Trans. Pattern Anal. Mach. Intell. **13**(6), 514 (1991)
19. Sapiro, G., Ringach, D.L.: Anisotropic diffusion of multivalued images with applications to color filtering. IEEE Trans. Image Process. **5**(11), 1582–1585 (1996)
20. Polzehl, J., Spokoiny, V.: Varying coefficient regression modeling. Preprint, Weierstrass Institute for Applied Analysis and Stochastics pp 818, 2003
21. Schavemaker, J.G.M., Reinders, M.J.T., Gerbrands, J.J., Backer, E.: Image sharpening by morphological filtering. Pattern Recogn. **33,** 997–1012 (2000)
22. Sethian, J.: Curvature and the evolution of fronts. Commun. Math. Phys. **101,** 487–499 (1985) MR0815197 (87d:58032)
23. Smith, S.M., Brady, J.M.: Susan–a new approach to low level image processing. Int. J. Comput. Vis. **23**(1), 45–78 (1997)
24. Tomasi, C., Manduchi, R.: Bilateral filtering for gray and color images. In: Sixth International Conference on Computer Vision pp. 839–46, 1998
25. Weickert, J.: Anisotropic Diffusion in Image Processing. Tuebner, Stuttgart (1998) MR1666943 (2000a:94003)
26. Yaroslavsky, L.P.: Digital Picture Processing—An Introduction. Springer, Berlin Heidelberg New York (1985) MR0818222 (88b:94001)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2264629 82C40 (35F20 35K57)
Bisi, M. [Bisi, Marzia] (I-PARM); **Desvillettes, L.** (F-ENSET-AM)
From reactive Boltzmann equations to reaction-diffusion systems. (English summary)
J. Stat. Phys. **124** (2006), *no. 2-4,* 881–912.

{A review for this item is in process.}

MR2258415 76D99 (35K57 76Z05)
Bernot, M. (F-ENSET-AM); **Caselles, V.** (E-POFA-T); **Morel, J. M.** (F-ENSET-AM)
Are there infinite irrigation trees? (English summary)
J. Math. Fluid Mech. **8** (2006), *no. 3,* 311–332.

{A review for this item is in process.}

References

1. A. Alvino, P. L. Lions and G. Trombetti, A remark on comparison results via symmetrisation, *Proceedings of the Royal Society of Edinburgh* **102A** (1986), 37–48. MR0837159 (88c:35007)
2. L. Ambrosio, *Lecture notes on optimal transport problems*, Preprint, 2000, available at http://cvgmt.sns.it. cf. MR2011032
3. A. Bejan, *Shape and Structure, from Engineering to Nature*, Cambridge University Press, 2000.
4. A. Bejan and M. R. Errera, Deterministic Tree Networks for Fluid Flow: Geometry for Minimal Flow Resistance between a Volume and one Point, *Fractals* **5(4)** (1997), 685–695.
5. V. Caselles and J. M. Morel, Irrigation, in: *Progress in Nonlinear Differential Equations and their Applications, VARMET, Trieste, June, 2001*, vol. 51, F. Tomarelli and G. Dal Maso (eds)., pp. 81–90, Birkhaüser, 2002. MR2197839
6. P. S. Dodds and D. H. Rothman, Unified view of scaling laws for river networks, *Phys. Rev. E* **59** (5) (1999), 4865–4877.
7. P. S. Dodds, D. H. Rothman and J. S. Weitz, Re-examination of the 3/4-law of metabolism, *Journal of Theoretical Biology* **209** (2001), 9–27.
8. R. E. Horton, Erosional Development of Streams and their Drainage Basins; Hydrophysical Approach to Quantitative Morphology, *Geol. Soc. Am. Bull.* **56** (1945), 275.
9. E. H. Lieb and M. Loss, *Analysis* Graduate Studies in Mathematics Vol. 14, 1997. MR1415616 (98b:00004)
10. F. Maddalena, J. M. Morel and S. Solimini, A variational model of irrigation patterns, *Interfaces and Free Boundaries* **5** (4) (2003), 391–416. MR2031464 (2004j:49065)
11. F. Morgan, *Geometric measure theory. A beginner's guide,* Academic Press, 1995. MR1326605 (96c:49001)
12. B. Mauroy, M. Filoche, E. R. Weibel and B. Sapoval, An optimal bronchial tree may be dangerous, *Nature* **427** 12th February, 2004, 633–636.
13. C. D. S. Murray, The physiological principle of minimum work. I, The vascular system and the cost of blood volume, *Proc. Nat. Acad. Sci. USA* **12** (1926), 207–214.
14. W. I. Newman, D. L. Turcotte and A. M. Gabrielov, Fractal Trees with Side Branching, *Fractals* **5** (4) (1997), 603–614.
15. I. Rodriguez-Iturbe and A. Rinaldo, *Fractal river basins, chance and self-organization*, Cambridge University Press, 2001.
16. B. Sapoval, *Universalités et Fractales,* Champs **466,** Flammarion, Paris, 1997.
17. G. Devillanova and S. Solimini, On the dimension of an irrigable measure, preprint.
18. A. N. Strahler, Quantitative Analysis of Watershed Geomorphology, *Am. Geophys. Un. Trans.* **38** (1957), 913.

19. G. Talenti, Elliptic equations and rearrangements *Ann. Scuola Norm. Sup. Pisa* **3** (1976). MR0601601 (58 #29170)
20. E. Tokunaga, Consideration on the Composition of Drainage Networks and their Evolution, *Geographical Rep. Tokyo Metro. Univ.* **13** (1978).
21. H. B. M. Uylings, Optimisation of diameters and bifurcation angles in lung and vascular tree structures, *Bull. Math. Biol.* **39,** (1977), 509–520.
22. Q. Xia, Optimal paths related to transport problems, *Commun. Contemp. Math.* **5,** (2003), 251–279. MR1966259 (2004a:90006)
23. Q. Xia, Interior regularity of optimal transport paths, *Calculus of Variations and Partial Differential Equations* **20** (3) (2004), 283–299. MR2062945 (2005a:90017)
24. Q. Xia, Boundary regularity of optimal transport paths, preprint.
25. G. B. West, The Origin of Universal Scaling Laws in Biology, *Physica A* **263** (1999), 104–113.
26. G. B. West, J. H. Brown and B. J. Enquist, A General Model for the Origin of Allometric Scaling Laws in Biology, *Science* **276** (4) (1997), 122–126.
27. G. B. West, J. H. Brown and B. J. Enquist, A general model for ontogenetic growth, *Nature* **413** (2001), 628–631.

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2246072 (2007k:94012) 94A08 (35A15 35K55 68U10 90C30)
Chan, Tony F. (1-UCLA); **Esedoḡlu, Selim** (1-MI); **Nikolova, Mila** (F-ENSET-AM)
Algorithms for finding global minimizers of image segmentation and denoising models. (English summary)
SIAM J. Appl. Math. **66** (2006), *no. 5,* 1632–1648 (*electronic*).

As it is stated by the authors in the first paragraph of this paper: “Image denoising and segmentation are two related, fundamental problems of computer vision. The goal of denoising is to remove noise and/or spurious details from a given, possibly corrupted, digital picture while maintaining essential features such as edges. The goal of segmentation is to divide the image into regions that belong to distinct objects in the depicted scene.”

In general, the mathematical models and computational algorithms for denoising are different from those for segmentation problems. However, before applying segmentation algorithms, it is customary to preprocess the image by using a denoising algorithm.

The denoising problem is challenging in itself because noise needs to be filtered out and, at the same time, edge information and other features must be preserved. Both edges and noise have high-frequency components, and, in most cases, they cannot be distinguished by using linear filtering. Thus, denoising models such as statistical filters and variational methods are nonlinear. The authors of the paper under review follow a variational approach to this problem. They look for the minima of the energy functional

$$E_m(u(\vec{x}),\lambda) := \int_{\mathbb{R}^2} |\nabla u(\vec{x})| d\vec{x} + \lambda \int_{\mathbb{R}^2} |u(\vec{x}) - \widetilde{u}(\vec{x})|^m d\vec{x},$$

where λ is the Lagrange multiplier, $\widetilde{u}(\cdot)$ is the noisy image, and $u(\cdot)$ is the clean image in the plane domain $\mathbb{R}^2$. The term $\int_{\mathbb{R}^2} |\nabla u(\vec{x})| d\vec{x}$ is called the total variation (TV) functional, and $\int_{\mathbb{R}^2} |u(\vec{x}) - \widetilde{u}(\vec{x})|^m d\vec{x}$ is the fidelity term in the L^m-norm. For instance, $m = 2$ has been studied for gray and color images by T. F. Chan, S. Osher and J. Shen [IEEE Trans. Image Process. **10** (2001), no. 2, 231–241; Zbl 1039.68778]. In the paper under review, however, the focus is on bi-level images with $m = 1$. The authors present an interesting procedure aimed at obtaining the global minimum of the energy functional.

Denoising can be also seen as a problem of signal (or function) estimation, where a signal (or image) corrupted by noise needs to be estimated. In this sense, it is worth noting the denoising algorithm based on soft-thresholding and wavelets by D. L. Donoho [IEEE Trans. Inform. Theory **41** (1995), no. 3, 613–627; MR1331258 (96b:94002)]. In that approach, different noise statistics can be naturally incorporated into the model.

On the other hand, the segmentation modeling approach of the paper under review is basically a two-phase piecewise constant Mumford-Shah segmentation model, or the equivalent by Chan and L. A. Vese [IEEE Trans. Image Process. **10** (2001), no. 2, 266–277] that is tailored to the case of bi-level images. These models are based on variational techniques and level sets, such as described in [Y.-H. R. Tsai and S. J. Osher, Acta Numer. **14** (2005), 509–573; MR2170510 (2006k:94010)] and many references therein. A faster version of this segmentation algorithm has been proposed by S. Gao and T. D. Bui [IEEE Trans. Image Process. **14** (2005), no. 10, 1537–1549]. In addition to tailoring the algorithm for bi-level images, the authors of the paper under review propose a method by which to restate the nonconvex optimization problem as a convex minimization problem with the corresponding global solution.

Reviewed by *Ramón M. Rodríguez-Dagnino*

References

1. S. Alliney, *Digital filters as absolute norm regularizers*, IEEE Trans. Signal Process., 40 (1992), pp. 1548–1562.
2. S. Alliney, *Recursive median filters of increasing order: A variational approach*, IEEE Trans. Signal Process., 44 (1996), pp. 1346–1354.
3. S. Alliney, *A property of the minimum vectors of a regularizing functional defined by means of the absolute norm*, IEEE Trans. Signal Process., 45 (1997), pp. 913–917.
4. L. Ambrosio and V. M. Tortorelli, *Approximation of functionals depending on jumps by elliptic functionals via Gamma convergence*, Comm. Pure Appl. Math., 43 (1990), pp. 999–1036. MR1075076 (92f:49003)
5. F. Catt'e, F. Dibos, and G. Koepfler, *A morphological scheme for mean curvature motion and applications to anisotropic diffusion and motion of level sets*, SIAM J. Numer. Anal., 32 (1995),

pp. 1895–1909. MR1360464 (96j:65105)
6. T. F. Chan and S. Esedo$\overline{glu}$, *Aspects of total variation regularized $L1$ function approxima- tion*, SIAM J. Appl. Math., 65 (2005), pp. 1817–1837. MR2177726 (2006k:94003)
7. T. F. Chan and L. A. Vese, *Active contours without edges*, IEEE Trans. Image Process., 10 (2001), pp. 266–277.
8. G. Dal Maso, *An Introduction to Γ-convergence*, Progr. Nonlinear Differential Equations Appl. 8, Birkh"auser Boston, Boston, MA, 1993. MR1201152 (94a:49001)
9. F. Dibos and G. Koepfler, *Global total variation minimization*, SIAM J. Numer. Anal., 37 (2000), pp. 646–664. MR1740764 (2001c:26016)
10. L. C. Evans and R. F. Gariepy, *Measure Theory and Fine Properties of Functions*, Studies in Advanced Mathematics, CRC Press, Boca Raton, FL, 1992. MR1158660 (93f:28001)
11. W. Fleming, W. Rishel, and R. Rishel, *An integral formula for total gradient variation*, Arch. Math., 11 (1960), pp. 218–222. MR0114892 (22 #5710)
12. E. Giusti, *Minimal Surfaces and Functions of Bounded Variation*, Monogr. Math. 80, Birkh"auser Verlag, Basel, 1984. MR0638362 (58 #30685)
13. J.-B. Hiriart-Urruty and C. Lemarechal, *Convex Analysis and Minimization Algorithms.* I. *Fundamentals*, Grundlehren Math. Wiss. 305, Springer-Verlag, New York, 1993. MR1261420 (95m:90001)
14. J.-B. Hiriart-Urruty and C. Lemarechal, *Convex Analysis and Minimization Algorithms.* II. *Advanced Theory and Bundle Methods*, Grundlehren Math. Wiss. 306, Springer-Verlag, New York, 1993. MR1295240 (95m:90002)
15. M. Kass, A. Witkin, and D. Terzopoulos, *Snakes: Active contour models*, Internat. J. Comput. Vision, 1 (1987), pp. 321–331.
16. L. Modica and S. Mortola, *Un esempio di Gamma-convergenza*, Boll. Un. Mat. Ital. B, 14 (1977), pp. 285–299. MR0445362 (56 #3704)
17. D. Mumford and J. Shah, *Optimal approximations by piecewise smooth functions and associated variational problems*, Comm. Pure Appl. Math., 42 (1989), pp. 577–685. MR0997568 (90g:49033)
18. M. Nikolova, *A variational approach to remove outliers and impulse noise*, J. Math. Imaging and Vision, 20 (2004), pp. 99–120. MR2049784 (2005b:94006)
19. M. Nikolova, *Minimizers of cost-functions involving nonsmooth data-fidelity terms. Application to the processing of outliers*, SIAM J. Numer. Anal., 40 (2002), pp. 965–994. MR1949401 (2003k:62170)
20. S. Osher and R. Fedkiw, *Level Set Methods and Dynamic Implicit Surfaces*, Appl. Math. Sci., 153, Springer-Verlag, New York, 2003. MR1939127 (2003j:65002)
21. S. Osher and J. Sethian, *Fronts propagating with curvature-dependent speed: Algorithms based on Hamilton–Jacobi formulations*, J. Comput. Phys., 79 (1988), pp. 12–49. MR0965860 (89h:80012)
22. L. Rudin, S. Osher, and E. Fatemi, *Nonlinear total variation based noise removal algorithms*, Phys. D, 60 (1992), pp. 259–268.
23. G. Strang, *$L1$ and $L\infty$ approximation of vector fields in the plane*, in Nonlinear Partial Differential Equations in Applied Science (Tokyo, 1982), North-Holland Math. Stud. 81, North-

Holland, Amsterdam, 1983, pp. 273–288. MR0730248 (85c:49010)
24. G. Strang, *Maximal flow through a domain*, Math. Programming, 26 (1983), pp. 123–143. MR0700642 (85e:90023)
25. D. Strong and T. F. Chan, *Edge-preserving and scale-dependent properties of total variation regularization*, Inverse Problems, 19 (2003), pp. S165–S187. MR2036526 (2005d:94014)
26. L. A. Vese and T. F. Chan, *A multiphase level set framework for image segmentation using the Mumford and Shah model*, Internat. J. Comput. Vision, 50 (2002), pp. 271–293.

Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.

MR2253559 (2007c:76052) 76M15 (65M50 76B15 76M25)
Fochesato, Christophe (F-ENSET-AM); **Dias, Frédéric** (F-ENSET-AM)
A fast method for nonlinear three-dimensional free-surface waves. (English summary)
Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci. **462** (2006), *no. 2073,* 2715–2735.

Summary: "An efficient numerical model for solving fully nonlinear potential flow equations with a free surface is presented. Like the code that was developed by S. T. Grilli, P. Guyenne and F. Dias [Internat. J. Numer. Methods Fluids **35** (2001), no. 7, 829–867; Zbl 1039.76043] it uses a high-order three-dimensional boundary-element method combined with mixed Eulerian-Lagrangian time updating, based on second-order explicit Taylor expansions with adaptive time-steps. Such methods are known to be accurate but expensive. The efficiency of the code has been greatly improved by introducing the fast multipole algorithm. By replacing every matrix-vector product of the iterative solver and avoiding the building of the influence matrix, this algorithm reduces the computing complexity from $O(N^2)$ to nearly $O(N)$, where N is the number of nodes on the boundary. The performance of the method is illustrated by the example of the overturning of a solitary wave over a three-dimensional sloping bottom. For this test case, the accelerated method is indeed much faster than the former one, even for quite coarse grids. For instance, a reduction of the complexity by a factor of six is obtained for $N = 6022$, for the same global accuracy. The acceleration of the code allows the study of more complex physical problems, and several examples are presented."

MR2243948 (2007h:35018) 35F20 (35-02 35B40 35K55 82C70)
Desvillettes, Laurent (F-ENSET-AM)
Hypocoercivity: the example of linear transport. (English summary)
Recent trends in partial differential equations, 33–53, *Contemp. Math.*, 409, *Amer. Math. Soc.*, *Providence, RI*, 2006.

Consider the evolution equation

$$\partial_t f(t) = Af(t)$$

and assume that it has the dissipative property

$$\partial_t H(f) = -D(f) \leq 0,$$

where $H(f), D(f)$ are functionals called entropy and dissipation, respectively. When

$$D(f) = 0 \Leftrightarrow Af = 0 \Leftrightarrow f = f_{\rm eq},$$

the equation is called coercive, where $f_{\rm eq}$ is a unique equilibrium under some conserved quantities. If we assume further that

$$D(f) \geq \Phi(H(f) - H(f_{\rm eq}))$$

for some positive function $\Phi(x) > 0$, $x \neq 0$, with $\Phi(0) = 0$, we can derive a convergence rate of $f(t) \to f_{\rm eq}$.

If the equation has the weaker property that

$$D(f) = 0 \quad \text{and} \quad Af = 0 \Rightarrow f = f_{\rm eq},$$

this property is called 'hypocoercivity'. In this case we can expect a convergence rate of $f(t) \to f_{\rm eq}$ if we construct an intermediate functional $K(f)$ such that

(1) $$D(f) \geq \Phi(K(f)),$$

(2) $$K(f) = 0 \Rightarrow f \in \mathcal{M} = \{f\colon\ D(f) = 0\},$$

and

(3) $$\frac{d^2(K(f))}{dt^2} + \Phi_2(K(f)) \geq \Phi_1(H(f) - H(f_{\rm eq})),$$

where Φ, Φ_1 and Φ_2 are appropriate positive functions. In this survey paper the author treats the linear transport equation

$$\partial_t f + v\partial_x f(t,x,v) = \overline{f} - f(t,x,v)$$

and explains the idea of hypocoercivity in detail, and further he gives a review on the applications in earlier works to the Fokker-Planck equation

$$Af = -v \cdot \nabla_x f + \nabla_x V \cdot \nabla_v f + \nabla_v \cdot (\nabla_v f + vf),$$

the linear Boltzmann equation

$$Af = -v \cdot \nabla_x f + x \cdot \nabla_v f - \rho_f M + f, \quad \rho_f = \int f dv,$$

and the nonlinear Boltzmann equation

$$Af = -v \cdot \nabla_x f + Q(f).$$

{For the entire collection see MR2238855 (2007b:35006)}

Reviewed by *Mitsuhiro Nakao*

MR2221319 (2007d:68170) 68U10 (94A08)

Strong, David M.; **Aujol, Jean-François** (F-ENSET-AM); **Chan, Tony F.** (1-UCLA)

Scale recognition, regularization parameter selection, and Meyer's G norm in total variation regularization. (English summary)

Multiscale Model. Simul. **5** (2006), *no. 1,* 273–303 (*electronic*).

Total variation (TV) based regularization as first proposed by Rudin, Osher and Fatemi (ROF) has been proven effective in preserving edges while recovering images. The unconstrained formulation of the ROF model reads as

$$\min_u aTV(u) + \|u - I\|_{L^2},$$

where I is the observed noisy image, u is the approximation to the true image $u_{\rm true}$, which is related to I by $I = u_{\rm true}+$ noise, and $TV(u) = \int |du|$ is the TV norm of u.

This paper investigates how TV regularization naturally recognizes the scale of individual features in images, and shows how perception of scale varies with increasing amounts of regularization, i.e., the increasing value of a in the above model. The notion of the scale is geometrically defined as the area of a feature divided by its perimeter in their early work.

The authors provide an algorithm for determining the minimum value of a that is large enough to result in removing all the features below a given scale threshold. This algorithm is an iterative process based on the standard bisection method and driven by the geometry of the image. Numerical results have shown its effectiveness.

This algorithm is also analyzed in this paper from the perspective of the Meyer G norm. The connection between the scale and the G norm helps give a more intuitive understanding of the G norm, and also gives some insight into this algorithm.

At the end of the paper, the authors explore two other applications of scale recognition. One is to determine at exactly which locations there is a feature below any given scale. This leads to some insight into multi-scale and scale-space effects of TV regularization. The other is to use the ability of determining scale at each discrete location in an image to examine the rate of loss of image features of various scales as a increasing. Numerical results are given to support their study.

Reviewed by Yun Mei Chen

References

1. R. Acar and C. Vogel, *Analysis of total variation penalty methods*, Inverse Problems, 10 (1994), pp. 1217–1229. MR1306801 (95i:65092)
2. H. Aly and E. Dubois, *Image up-sampling using total-variation regularization with a new observation model*, IEEE Trans. Image Process., 14 (2005), pp. 1647–1659. MR2147827 (2006d:94007)
3. L. Ambrosio, N. Fusco, and D. Pallara, *Functions of Bounded Variation and Free Discon- tinuity Problems*, Oxford Math. Monogr., Oxford University Press, New York, 2000. MR1857292 (2003a:49002)
4. F. Andreu, V. Caselles, J. Diaz, and J. Mazon, *Some qualitative properties for the total variation flow*, J. Funct. Anal., 188 (2002), pp. 516–547. MR1883415 (2003a:35092)
5. G. Aubert and J.-F. Aujol, *Modeling very oscillating signals. Application to image processing*, Appl. Math. Optim., 51 (2005), pp. 163–182. MR2117231 (2006k:49112)
6. J.-F. Aujol, G. Aubert, L. Blanc-F'eraud, and A. Chambolle, *Image decomposition into a bounded variation component and an oscillating component*, J. Math. Imaging Vision, 22 (2005), pp. 71–88. MR2138585 (2005m:68219)
7. J.-F. Aujol and A. Chambolle, *Dual norms and image decomposition models*, Internat. J. Comput. Vision, 63 (2005), pp. 85–104.
8. J.-F. Aujol, G. Gilboa, T. Chan, and S. Osher, *Structure-texture image decomposition – modeling, algorithms, and parameter selection*, Internat. J. Comput. Vision, 67 (2006), pp. 111–113.
9. G. Bellettini, V. Caselles, and M. Novaga, *The total variation flow in* **R***n*, J. Differential Equations, 184 (2002), pp. 475–525. MR1929886 (2003g:35105)
10. P. Blomgren and T. Chan, *Color TV: Total variation methods for restoration of vector- valued images*, IEEE Trans. Image Process., 7 (1998), pp. 304–309.
11. T. Brox and J. Weickert, *A TV flow based local scale measure for texture discrimination*, in Proceedings of the 8th European Conference on Computer Vision, Vol. 2, Prague, Czech Republic, 2004, pp. 578–590.
12. A. Chambolle, *An algorithm for total variation minimization and applications*, J. Math. Imaging Vision, 20 (2004), pp. 89–97. MR2049783 (2005m:49058)
13. A. Chambolle and P.-L. Lions, *Image recovery via total variation minimization and related problems*, Numer. Math., 76 (1997), pp. 167–188. MR1440119 (98c:65099)
14. T. F. Chan, G. H. Golub, and P. Mulet, *A nonlinear primal-dual method for total variation-based image restoration*, SIAM J. Sci. Comput., 20 (1999), pp. 1964–1977. MR1694649 (2000c:65038)
15. T. F. Chan and J. Shen, *Mathematical models for local nontexture inpaintings*, SIAM J. Appl. Math., 62 (2002), pp. 1019–1043. MR1897733 (2003f:65110)
16. T. Chan and C.-K. Wong, *Total variation blind deconvolution*, IEEE Trans. Image Process., 7 (1998), pp. 370–375.

17. Q. Chang and I-L. Chern, *Acceleration methods for total variation-based image denoising*, SIAM J. Sci. Comput., 25 (2003), pp. 982–994. MR2046121 (2005f:65064)
18. D. C. Dobson and F. Santosa, *Recovery of blocky images from noisy and blurred data*, SIAM J. Appl. Math., 56 (1996), pp. 1181–1198. MR1398414 (97c:68166)
19. D. C. Dobson and C. R. Vogel, *Convergence of an iterative method for total variation denoising*, SIAM J. Numer. Anal., 34 (1997), pp. 1779–1791. MR1472196 (99a:65081)
20. G. Gilboa, N. Sochen, and Y. Y. Zeevi, *PDE-based denoising of complex scenes using a spatially-varying fidelity term*, in Proceedings of the IEEE International Conference on Image Processing, Vol. 1, Barcelona, Spain, 2003, pp. 865–868.
21. Y. Gousseau and J.-M. Morel, *Are natural images of bounded variation?*, SIAM J. Math. Anal., 33 (2001), pp. 634–648. MR1871413 (2003a:94009)
22. Y. Meyer, *Oscillating Patterns in Image Processing and in Some Nonlinear Evolution Equations: The Fifteenth Dean Jacqueline B. Lewis Memorial Lectures*, AMS, Providence, RI, 2001. MR1852741 (2002j:43001)
23. S. Osher, A. Sol'e, and L. Vese, *Image decomposition and restoration using total variation minimization and the H –1 norm*, Multiscale Model. Simul., 1 (2003), pp. 349–370. MR2030155 (2004k:49004)
24. P. Perona and J. Malik, *Scale space and edge detection using anisotropic diffusion*, IEEE Transactions on Pattern Analysis Machine Intelligence, 12 (1990), pp. 629–639.
25. E. Radmoser, O. Scherzer, and J. Weickert, *Scale-space properties of regularization meth- ods*, in Scale-Space Theories in Computer Vision, Lecture Notes in Comput. Sci. 1682, M. Nielsen, P. Johansen, O. F. Olsen, and J. Weickert, eds., Springer, Berlin, 1999, pp. 211–222.
26. L. Rudin, S. Osher, and E. Fatemi, *Nonlinear total variation based noise removal algorithms*, Phys. D, 60 (1992), pp. 259–268.
27. G. Strang, *$L1$ and $L\infty$ approximation of vector fields in the plane*, in Nonlinear Partial Differential Equations in Applied Science, North–Holland, Amsterdam, 1983, pp. 273–288. MR0730248 (85c:49010)
28. D. Strong, *Adaptive Total Variation Minimizing Image Restoration*, Ph.D. thesis, UCLA Math Department, CAM Report 97-38, University of California, Los Angeles, CA, 1997.
29. D. Strong and T. Chan, *Edge-preserving and scale-dependent properties of total variation regularization*, Inverse Problems, 19 (2003), pp. 165–187. MR2036526 (2005d:94014)
30. E. Tadmor, S. Nezzar, and L. Vese, *A multiscale image representation using hierarchical* (BV, $L2$) *decompositions*, Multiscale Model. Simul., 2 (2004), pp. 554–579. MR2113170 (2005h:68163)
31. A. Tikhonov and V. Arsenin, *Solution of Ill-Posed Problems*, Winston and Sons, Washington, DC, 1977. MR0455365 (56 #13604)
32. L. Vese and S. Osher, *The Level Set Method Links Active Contours, Mumford-Shah Seg- mentation, and Total Variation Restoration*, UCLA CAM Report 02-05, University of California, Los Angeles, CA, 2002, http://www.math.ucla.edu/applied/cam/index.html.
33. C. R. Vogel, *Computational Methods for Inverse Problems*, SIAM, Philadelphia, 2002. MR1928831 (2003i:65004)
34. C. R. Vogel and M. E. Oman, *Iterative methods for total variation denoising*, SIAM J. Sci.

Comput., 17 (1996), pp. 227–238. MR1375276

Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.

MR2231282 (2007e:35024) 35F20 (35B40 76P05 82C40)

Desvillettes, L. (F-ENSET-AM)

About the large time behavior of dissipative equations when a priori bounds are slowly growing.

‘‘*WASCOM* 2005’’—13*th Conference on Waves and Stability in Continuous Media*, 193–203, *World Sci. Publ., Hackensack, NJ*, 2006.

Summary: “We are interested in obtaining explicit rates of convergence toward the equilibrium for equations having a Lyapunov functional (entropy). We assume that the dissipation of entropy dominates the entropy itself. However, instead of being independent of time, we only suppose that the rate of this domination deteriorates slightly when the time goes to infinity. Such a situation was described first by Toscani and Villani in the context of the Boltzmann equation (and its variants when grazing collisions are predominant). We show here how the estimate of convergence toward equilibrium obtained there can be used to establish new (global in time) a priori estimates for the equation under study, which, in turn, sometimes enables the rate of convergence toward equilibrium to be found. Examples of applications of these ideas are taken from works in collaboration with C. Mouhot [“Large time behavior of the a priori bounds for the solutions to the spatially homogeneous Boltzman equation with soft potentials”, Preprint No. 2005-02, Centre Math. Leurs Appl. (CMLA), Cachan, 2005] and K. Fellner [“Entropy methods for reaction-diffusion equations: degenerate diffusion and slowly growing a priori bounds”, Preprint No. 2005-19, CMLA, Cachan, 2005], respectively for homogeneous kinetic equations and reaction-diffusion systems.”

{For the entire collection see MR2227253 (2006m:00012)}

MR2227097 (2007e:68064) 68U10 (53C20 68T10 68T45 92C50)

Miller, Michael I. (1-JHOP-CIS); **Trouvé, Alain** (F-ENSET-AM);
Younes, Laurent (1-JHOP-CIS)

Geodesic shooting for computational anatomy. (English summary)

J. Math. Imaging Vision **24** (2006), *no. 2,* 209–228.

Summary: "Studying large deformations using a Riemannian approach has been an efficient way to generate metrics between deformable objects, and to provide accurate, nonambiguous and smooth matchings between images. In this paper, we study the geodesics of such large deformation diffeomorphisms, and more precisely, introduce a fundamental property that they satisfy, namely the conservation of momentum. This property allows us to generate and store complex deformations with the help of one initial 'momentum' which serves as the initial state of a differential equation in the group of diffeomorphisms. Moreover, it is shown that this momentum can also be used to describe a deformation of given visual structures, like points, contours or images, with the same dimension as the described object, as a consequence of the normal momentum constraint we introduce."

Reviewed by *Tian Zhou Xu*

MR2218728 (2006m:49047) 49K40 (49J52 90C31 94A08 94A12)

Durand, S. [Durand, Sylvain] (F-PCRD-LFD);
Nikolova, M. [Nikolova, Mila] (F-ENSET-AM)

Stability of the minimizers of least squares with a non-convex regularization. II. Global behavior. (English summary)

Appl. Math. Optim. **53** (2006), *no. 3,* 259–277.

The authors consider a problem of minimizing an objective function which is the sum of a quadratic data-fidelity term and a regularization term. They study the stability of the minimizers of such an objective function when the regularization term is piecewise $\mathcal{C}^m$ (with $m \geq 2$) and non-convex. In the first part of their work [Appl. Math. Optim. **53** (2006), no. 2, 185–208; MR2172785], local results were established. In this second part they derive corresponding results for global stability. Their results imply that the data domain contains an open, dense subset such that for each data point therein the objective function has finitely many local minimizers and a unique global minimizer.

Reviewed by *Zili Wu*

References

1. J. E. Besag, On the statistical analysis of dirty pictures (with discussion), Journal of the Royal Statistical Society B, 48 (1986), 259–302. MR0876840 (88c:68079)
2. M. Black, G. Sapiro, D. Marimont, and D. Heeger, Robust anisotropic diffusion, IEEE Transactions on Image Processing, 7 (1998), 421–432.
3. J. Bonnans and A. Shapiro, Perturbation Analysis of Optimization Problems, Springer-Verlag, New York, 2000. MR1756264 (2001g:90003)
4. C. Bouman and K. Sauer, A generalized Gaussian image model for edge-preserving map estimation, IEEE Transactions on Image Processing, 2 (1993), 296–310.
5. P.G. Ciarlet, Introduction à l'analyse numérique matricielle et à l'optimisation, Collection mathématiques appliquées pour la maîtrise, Masson, Paris, 1990.
6. A. L. Dontchev and T. Zollezi, Well-Posed Optimization Problems, Springer-Verlag, New York, 1993. MR1239439 (95a:49002)
7. S. Durand and M. Nikolova, Stability of the minimizers of least squares with a non-convex regularization. Part I: local behavior, Applied Mathematics & Optimization, 53 (2206), 185–208. MR2172785 (2006k:90132)
8. S. Geman and D. Geman, Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images, IEEE Transactions on Pattern Analysis and Machine Intelligence, PAMI-6 (1984), 721–741.
9. S. Geman and D. E. McClure, Statistical methods for tomographic image reconstruction, in Proceedings of the 46th Session of the ISI, Bulletin of the ISI, vol. 52, 1987, pp. 22–26. MR1027188
10. P. J. Green, Bayesian reconstructions from emission tomography data using a modified em algorithm. IEEE Transactions on Medical Imaging, MI-9 (1990), 84–93.
11. P. Perona and J. Malik, Scale-space and edge detection using anisotropic diffusion, IEEE Transactions on Pattern Analysis and Machine Intelligence, PAMI-12 (1990), 629–639.
12. R. T. Rockafellar and J. B. Wets, Variational Analysis, Springer-Verlag, New York, 1997.
13. L. Rudin, S. Osher, and C. Fatemi, Nonlinear total variation based noise removal algorithm, Physica, 60 D (1992), 259–268.
14. S. Teboul, L. Blanc-Féraud, G. Aubert, and M. Barlaud, Variational approach for edge-preserving regularization using coupled pde's, IEEE Transactions on Image Processing, 7 (1998), 387–397.

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2217853 (2007b:35192) 35K57 (35B40 92E20)
Desvillettes, Laurent (F-ENSET-AM); **Fellner, Klemens** (A-WIENM)
Exponential decay toward equilibrium via entropy methods for reaction-diffusion equations. (English summary)
J. Math. Anal. Appl. **319** (2006), *no. 1,* 157–176.

An explicit exponential decay rate toward an equilibrium state is obtained for bounded solutions of two problems. The first one consists of the system

$$\partial_t a - d_a \Delta_x a = -2(a^2 - b),$$
$$\partial_t b - d_b \Delta_x b = a^2 - b$$

with homogeneous Neumann boundary conditions and non-negative initial conditions, where $x \in \Omega \subset \mathbb{R}^N$ and $t > 0$. The second problem is

$$\partial_t a - d_a \Delta_x a = -ab + c,$$
$$\partial_t b - d_b \Delta_x b = -ab + c,$$
$$\partial_t c - d_c \Delta_x c = ab - c$$

with the same kind of boundary and initial conditions, where $x \in \Omega \subset \mathbb{R}^N$ and $t > 0$.

In all these equations, d_a, d_b and d_c are constant diffusion rates. The decay rates toward constant steady states are obtained via entropy methods, where, for the first problem, the entropy functional takes the form

$$E(a,b) = \int_\Omega a(\ln a - 1) + b(\ln b - 1)dx$$

and the entropy dissipation is given by

$$D(a,b) = d_a \int_\Omega \frac{|\nabla a|^2}{a} + d_b \frac{|\nabla b|^2}{b} + (a^2 - b)\ln \frac{a^2}{b} dx.$$

Reviewed by *Arnaud Rougirel*

References

1. H. Amann, Global existence for semilinear parabolic problems, J. Reine Angew. Math. 360 (1985) 47–83. MR0799657 (87b:35089)
2. A. Barabanova, On the global existence of solutions of a reaction-diffusion equation with exponential nonlinearity, Proc. Amer. Math. Soc. 122 (1994) 827–831. MR1207533 (95a:35072)
3. M. Bisi, L. Desvillettes, From reactive Boltzmann equations to reaction-diffusion systems, preprint 2005–11 of the CMLA, ENS de Cachan, 2005. cf. MR 2007m:82084
4. D. Bothe, D. Hilhorst, A reaction-diffusion system with fast reversible reaction, J. Math. Anal. Appl. 286 (2003) 125–135. MR2009623 (2005h:35145)
5. M. Caceres, J. Carrillo, G. Toscani, Long-time behavior for a nonlinear fourth order parabolic equation, Trans. Amer. Math. Soc. 357 (2005) 1161–1175 (electronic). MR2110436 (2005k:35188)
6. M. Caceres, J. Carrillo, T. Goudon, Equilibration rate for the linear inhomogeneous relaxation-time Boltzmann equation for charged particles, Comm. Partial Differential Equations 28 (2003)

969–989. MR1986057 (2004g:82111)
7. E. Conway, D. Hoff, J. Smoller, Large time behaviour of solutions of systems of nonlinear reaction-diffusion equations, SIAM J. Appl. Math. 35 (1978) 1–16. MR0486955 (58 #6637)
8. I. Csiszár, Eine informationstheoretische Ungleichung und ihre Anwendung auf den Beweis von Markoffschen Ketten, Magyar Tud. Akad. Mat. Kutató Int. Közl. 8 (1963) 85–108. MR0164374 (29 #1671)
9. J. Carrillo, J.L. Vazquez, Fine asymptotics for fast diffusion equations, Comm. Partial Differential Equations 28 (2003) 1023–1056. MR1986060 (2004a:35118)
10. M. Del Pino, J. Dolbeault, Best constants for Gagliardo-Nirenberg inequalities and applications to nonlinear diffusions, J. Math. Pures Appl. 81 (2002) 847–875. MR1940370 (2003h:35051)
11. A. De Masi, E. Presutti, Mathematical Methods for Hydrodynamic Limits, Springer, Berlin, 1991. MR1175626 (93k:60234)
12. L. Desvillettes, K. Fellner, Entropy methods for reaction-diffusion equations: degenerate diffusion and slowly growing a-priori bounds, preprint HYKE2005–103, available at www.hyke.org.
13. L. Desvillettes, C. Villani, On the spatially homogeneous Landau equation for hard potentials. II. H-theorem and applications, Comm. Partial Differential Equations 25 (2000) 261–298. MR1737548 (2001c:82066)
14. L. Desvillettes, C. Villani, On the trend to global equilibrium in spatially inhomogeneous entropy-dissipating systems: the linear Fokker-Planck equation, Comm. Pure Appl. Math. 54 (2001) 1–42. MR1787105 (2001h:82079)
15. L. Desvillettes, C. Villani, On the trend to global equilibrium for spatially inhomogeneous kinetic systems: the Boltzmann equation, Invent. Math. 159 (2005) 245–316. MR2116276 (2005j:82070)
16. P. Érdi, J. Tóth, Mathematical models of chemical reactions, in: Nonlinear Science: Theory and Applications, Manchester Univ. Press, 1989. MR0981593 (90h:92037)
17. K. Fellner, L. Neumann, C. Schmeiser, Convergence to global equilibrium for spatially inhomogeneous kinetic models of non-micro-reversible processes, Monatsh. Math. 141 (2004) 289–299. MR2053654 (2005f:82116)
18. W. Feng, Coupled system of reaction-diffusion equations and applications in carrier facilitated diffusion, Nonlinear Anal. 17 (1991) 285–311. MR1120978 (92j:92009)
19. P.C. Fife, Mathematical Aspects of Reacting and Diffusing Systems, Lecture Notes in Biomath., vol. 28, Springer, Berlin, 1979. MR0527914 (80g:35001)
20. W.E. Fitzgibbon, J. Morgan, R. Sanders, Global existence and boundedness for a class of inhomogeneous semilinear parabolic systems, Nonlinear Anal. 19 (1992) 885–899. MR1190873 (94a:35057)
21. W. Fitzgibbon, S. Hollis, J. Morgan, Stability and Lyapunov functions for reaction-diffusion systems, SIAM J. Math. Anal. 28 (1997) 595–610. MR1443610 (98d:35107)
22. K. Gröger, Free energy estimates and asymptotic behaviour of reaction-diffusion processes, preprint 20, Institut für Angewandte Analysis und Stochastik, Berlin (1992).
23. A. Glitzky, K. Gröger, R. Hünlich, Free energy and dissipation rate for reaction-diffusion processes of electrically charged species, Appl. Anal. 60 (1996) 201–217. MR1645555

24. A. Glitzky, R. Hünlich, Energetic estimates and asymptotics for electro-reaction-diffusion systems, Z. Angew. Math. Mech. 77 (1997) 823–832. MR1488954 (98k:35099)
25. A. Haraux, A. Youkana, On a result of K. Masuda concerning reaction-diffusion equations, Tôhoku Math. J. 4 (1988) 159–163.
26. S. Hollis, R. Martin, M. Pierre, Global existence and boundedness in reaction-diffusion systems, SIAM J. Math. Appl. 18 (1987) 744–761. MR0883566 (88c:35081)
27. H. Hoshino, Y. Yamada, Asymptotic behavior of global solutions for some reaction-diffusion systems, Nonlinear Anal. 23 (1994) 639–650. MR1297283 (95g:35091)
28. Y.I. Kanel, Cauchy's problem for semilinear parabolic equations with balance conditions, Differ. Uravn. 20 (1984) 1753–1760. English transl.: Differential Equations 20 (1984) 1260–1266. MR0767886 (86e:35069)
29. Y.I. Kanel, M. Kirane, Global solutions of reaction-diffusion systems with a balance law and nonlinearities of exponential growth, J. Differential Equations 165 (2000) 24–41. MR1771787 (2001g:35127)
30. M. Kirane, On stabilization of solutions of the system of parabolic differential equations describing the kinetics of an autocatalytic reversible chemical reaction, Bull. Inst. Mat. Acad. Sinica 18 (1990) 369–377. MR1104954 (92e:35083)
31. S. Kullback, Information Theory and Statistics, Wiley, 1959. MR0103557 (21 #2325)
32. R.H. Martin, M. Pierre, Nonlinear reaction-diffusion systems, in: W.F. Ames, C. Rogers (Eds.), Nonlinear Equations in Applied Sciences, in: Math. Sci. Eng., vol. 185, Academic Press, New York, 1991.
33. K. Masuda, On the global existence and asymptotic behavior of solution of reaction-diffusion equations, Hokkaido Math. J. 12 (1983) 360–370. MR0719974 (84m:35017)
34. J. Morgan, Global existence for semilinear parabolic systems, SIAM J. Math. Anal. 20 (1989) 1128–1144. MR1009350 (90h:35121)
35. F. Rothe, Global Solutions of Reaction-Diffusion Systems, Lecture Notes in Math., Springer, Berlin, 1984. MR0755878 (86d:35071)
36. J. Smoller, Shock Waves and Reaction-Diffusion Equations, Grundlehren Math. Wiss., vol. 258, Springer, New York, 1983. MR0688146 (84d:35002)
37. R. Spigler, D.H. Zanette, Reaction-diffusion models from the Fokker-Planck formulation of chemical processes, IMA J. Appl. Math. 49 (1992) 217–229. MR1198791 (93i:92021)
38. R. Spigler, D.H. Zanette, A BGK model for chemical processes: the reaction-diffusion approximation, Math. Models Methods Appl. Sci. 4 (1994) 35–47. MR1259201 (94k:82101)
39. A.-S. Sznitman, Propagation of chaos for a system of annihilating Brownian spheres, Comm. Pure Appl. Math. 40 (1987) 663–690. MR0910949 (89h:60142)
40. G. Toscani, C. Villani, Sharp entropy dissipation bounds and explicit rate of trend to equilibrium for the spatially homogeneous Boltzmann equation, Comm. Math. Phys. 203 (1999) 667–706. MR1700142 (2000e:82039)
41. G. Toscani, C. Villani, On the trend to equilibrium for some dissipative systems with slowly increasing a priori bounds, J. Statist. Phys. 98 (2000) 1279–1309. MR1751701 (2001g:82069)
42. C. Villani, Cercignani's conjecture is sometimes true and always almost true, Comm. Math. Phys. 234 (2003) 455–490. MR1964379 (2004b:82048)

43. A. Volpert, V.A. Volpert, V.A. Volpert, Traveling Wave Solutions of Parabolic Systems, Transl. Math. Monogr., vol. 140, Amer. Math. Soc., Providence, RI, 1994. MR1297766 (96c:35092)
44. G.F. Webb, Theory of Nonlinear Age-Dependent Population Dynamics, Monogr. Textbooks Pure Appl. Math., vol. 89, Dekker, New York, 1985. MR0772205 (86e:92032)
45. T. Zelenyak, Stabilization of solutions to boundary value problems for second order parabolic equations in one space variable, Differ. Uravn. 4 (1968) 34–45. MR0223758 (36 #6806)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2214160 (2006k:35226) 35Q35 (35B30 76N10 76T10)

Baranger, Céline; **Desvillettes, Laurent** (F-ENSET-AM)

Coupling Euler and Vlasov equations in the context of sprays: the local-in-time, classical solutions. (English summary)

J. Hyperbolic Differ. Equ. **3** (2006), *no. 1,* 1–26.

Summary: "Sprays are complex flows which consist of liquid droplets surrounded by a gas. They can be modeled by introducing a system coupling a kinetic equation (for the droplets) of Vlasov type and a (Euler-like) fluid equation for the gas. In this paper, we prove that, for the so-called thin sprays, this coupled model is well-posed, in the sense that existence and uniqueness of classical solutions hold for small time, provided the initial data are sufficiently smooth and their support have suitable properties."

Reviewed by *Xiaoming Wang*

References

1. A. A. Amsden, P. J. O'Rourke and T. D. Butler, Kiva-II, a computer program for chemical reactive flows with sprays, Technical Report, Los Alamos National Laboratory (1989).
2. R. Caflisch and G. Papanicolaou, Dynamic theory of suspensions with brownian effects, *SIAM J. Appl. Math.* **43** (1983) 885–906. MR0709743 (84k:82071)
3. J. F. Clouet and K. Domelevo, Solutions of a kinetic stochastic equation modeling a spray in a turbulent gas flow, *Math. Models Methods Appl. Sci.* **7** (1997) 239–263. MR1440608 (98m:76095)
4. K. Domelevo, Long time behaviour for a kinetic modelling of two-phase flows with thin sprays and point particles, Preprint de TMR-project "Asymptotics Methods in Kinetic Theory" (2001).
5. K. Domelevo and J.-M. Roquejoffre, Existence and stability of travelling waves solutions in a kinetic model of two phase flows, *Commun. Partial Differential Equations* **24**(1–2) (1999)

61–108. MR1671989 (2000b:35276)
6. K. Domelevo and M.-H. Vignal, Limites visqueuses pour des systèmes de type Fokker-Planck-Burgers unidimensionnels, *C. R. Acad. Sci. Paris Sér. I Math.* **332**(9) (2001) 863–868. MR1836101 (2002c:76099)
7. T. Goudon, Asymptotic problems for a kinetic model of two-phase flow, *Proc. Roy. Soc. Edinburgh Sect. A* **131**(6) (2001) 1371–1384. MR1869641 (2002k:76130)
8. K. Hamdache, Global existence and large time behaviour of solutions for the Vlasov-Stokes equations, *Japan J. Indust. Appl. Math.* **15**(1) (1998) 51–74. MR1610309 (99f:76128)
9. B. L. Keyfitz, R. Sanders and M. Sever, Lack of hyperbolicity in the two-fluid model for two-phase incompressible flow, *Discrete Contin. Dynam. Syst.* **B3** (2003) 541–563. MR2036001 (2004k:76123)
10. A. Majda, *Compressible Fluid Flows and Systems of Conservation Laws in Several Space Variables* (Springer-Verlag, 1984). MR0748308 (85e:35077)
11. R. Motte, A numerical method for solving particle-fluid equations, in *Proc. Conf. Trends in Numerical and Physical Modeling for Industrial Multiphase Flows*, Cargèse, France (2000).
12. M. Ndjinga, A. Kumbaro, F. De Vuyst and P. Laurent-Gencoux, Influence of the interfacial forces on the hyperbolicity of the two-fluid model, in *5th Int. Symp. Multiphase Flow, Heat Mass Transfer and Energy Conversion*, Xi'an, China (July 3–6 2005).
13. P. J. O'Rourke, Collective drop effects on vaporizing liquid sprays, PhD thesis, Los Alamos National Laboratory (1981).
14. K. PfaffelMoser, Global classical solutions of the Vlasov-Poisson system in three dimensions for general initial data, *J. Differential Equations* **95**(2) (1992) 281–303. MR1165424 (93d:35170)
15. D. Ramos, Quelques résultats mathématiques et simulations num ériques d'écoulements régis par des modèles bifluides, PhD thesis, Ecole Normale Supérieure de Cachan (2000).
16. W. E. Ranz and W. R. Marshall, Evaporization from drops, Part I-II, *Chem. Eng. Prog.* **48**(3) (1952) 141–180.
17. D. Serre, *Systèmes de lois de Conservation. I* (Fondations. Diderot Editeur, Paris, 1996). MR1459988 (99b:35139)
18. F. A. Williams, *Combustion Theory* (Benjamin Cummings, 1985).

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

MR2211432 (2007h:94008) 94A08 (49M15 49N45 65K10)

Fu, Haoying (1-PAS-CE); **Ng, Michael K.** (PRC-HK); **Nikolova, Mila** (F-ENSET-AM); **Barlow, Jesse L.** (1-PAS-CE)

Efficient minimization methods of mixed $l2$-$l1$ and $l1$-$l1$ norms for image restoration. (English summary)

SIAM J. Sci. Comput. **27** (2006), *no. 6,* 1881–1902 (*electronic*).

Summary: "Image restoration problems are often solved by finding the minimizer of a suitable objective function. Usually this function consists of a data-fitting term and a regularization term. For the least squares solution, both the data-fitting and the regularization terms are in the $l2$ norm. In this paper, we consider the least absolute deviation (LAD) solution and the least mixed norm (LMN) solution. For the LAD solution, both the data-fitting and the regularization terms are in the $l1$ norm. For the LMN solution, the regularization term is in the $l1$ norm but the data-fitting term is in the $l2$ norm. Since images often have nonnegative intensity values, the proposed algorithms provide the option of taking into account the nonnegativity constraint. The LMN and LAD solutions are formulated as the solution to a linear or quadratic programming problem which is solved by interior point methods. At each iteration of the interior point method, a structured linear system must be solved. The preconditioned conjugate gradient method with factorized sparse inverse preconditioners is employed to solve such structured inner systems. Experimental results are used to demonstrate the effectiveness of our approach. We also show the quality of the restored images, using the minimization of mixed $l2$-$l1$ and $l1$-$l1$ norms, is better than that using only the $l2$ norm."

References

1. N. Abdelmalek, *Restoration of images with missing high frequency components using quadratic programming*, Appl. Opt., 22 (1983), pp. 2182–2188.
2. N. Abdelmalek and T. Kasvand, *Digital image restoration using quadratic programming*, Appl. Opt., 19 (1980), pp. 3407–3415.
3. N. Abdelmalek and N. Otsu, *Restoration of images with missing high-frequency components by minimizing the* 1 *norm of the solution vector*, Appl. Opt., 24 (1985), pp. 1415–1420.
4. S. Alliney, *Digital filters as absolute norm regularizers*, IEEE Trans. Signal Process., 40 (1992), pp. 1548–1562.
5. S. Alliney, *A property of the minimum vectors of a regularizing functional defined by means of absolute norm*, IEEE Trans. Signal Process., 45 (1997), pp. 913–917.
6. S. Alliney and S. Ruzinsky, *An algorithm for the minimization of mixed* 1 *and* 2 *norms with application to Bayesian estimation*, IEEE Trans. Signal Process., 42 (1994), pp. 618–627.
7. M. Benzi and G. H. Golub, *A preconditioner for generalized saddle point problems*, SIAM J. Matrix Anal. Appl., 26 (2004), pp. 20–41. MR2112850 (2005j:65026)
8. M. Benzi, C. Meyer, and M. Tuma, *A sparse approximate inverse preconditioner for the conjugate gradient method*, SIAM J. Sci. Comput., 17 (1996), pp. 1135–1149. MR1404865 (97d:65025)
9. P. Bloomfield and W. Steiger, *Least Absolute Deviations: Theory, Applications, and Al-*

gorithms, Birkh”auser, Boston, 1983. MR0748483 (86a:62005)

10. L. Canales, *L1 Norm Spectral Estimation*, Technical report, Stanford University, Stanford Exploration Project, http://sepwww.stanford.edu/oldreports/sep07/sp07 16.pdf (1975).
11. A. Chambolle, *An algorithm for total variation minimization and applications*, J. Math. Imaging Vision, 20 (2004), pp. 89–97. MR2049783 (2005m:49058)
12. T. Chan and S. Esedoglu, *Aspects of Total Variation Regularized* 1 *Function Approximation*, Technical report, University of California at Los Angeles, 2004.
13. G. Chavent and K. Kunisch, *Regularization of linear least squares problems by total bounded variation*, ESAIM: Control Optim. Calc. Var., 2 (1997), pp. 359–376. MR1483764 (99b:65071)
14. P. Combettes, *An adaptive level set method for nondifferentiable constrained image recovery*, IEEE Trans. Image Process., 11 (2002), pp. 1295–1304. MR1961417 (2004c:94009)
15. A. Dax, *A row relaxation method for large* 1 *problems*, Linear Algebra Appl., 156 (1991), pp. 793–818. MR1113170 (92e:90049)
16. A. Dax, *On regularized least norm problems*, SIAM J. Optim., 2 (1992), pp. 602–618. MR1186164 (93k:65055)
17. A. Dax, *On row-relaxation methods for large constrained least squares problems*, SIAM J. Sci. Comput., 14 (1993), pp. 570–584. MR1214769 (93m:65081)
18. A. Dax, *A row relaxation method for large p least norm problems*, Numer. Linear Algebra Appl., 1 (1994), pp. 247–263. MR1299681 (95j:90051)
19. D. Etter, D. Kuncicky, and D. Hull, *Introduction to Matlab* 6, Prentice-Hall, Upper Saddle River, NJ, 2002.
20. H. Fu and J. Barlow, *A regularized structured total least squares algorithm for high resolution image reconstruction*, Linear Algebra Appl., 391 (2004), pp. 75–98. MR2094604 (2005i:65062)
21. A. Guitton and D. J. Verschuur, *Adaptive subtraction of multiples using the* 1*-norm*, Geophys. Prospecting, 52 (2004), pp. 1–27.
22. M. Hanke, J. Nagy, and C. Vogel, *Quasi-Newton approach to nonnegative image restora- tions*, Linear Algebra Appl., 316 (2000), pp. 223–236. MR1783307 (2001e:94003)
23. M. Hinterm”uller and K. Kunisch, *Total bounded variation regularization as a bilaterally constrained optimization problem*, SIAM J. Appl. Math., 64 (2004), pp. 1311–1333. MR2068672 (2005b:49062)
24. K. Ito and K. Kunisch, *An active set strategy based on the augmented Lagrangian formulation for image restoration*, ESAIM: Math. Model. Numer. Anal., 33 (1999), pp. 1–21. MR1685741 (2000m:49044)
25. T. K”arkk”ainen, K. Kunisch, and K. Majava, *Denoising of smooth images using L*1*-fitting*, Computing, 74 (2005), pp. 353–376. MR2149344 (2006f:94005)
26. Q. Ke and T. Kanade, *Robust Subspace Computation Using* 1 *Norm*, Technical report, School of Computer Science, Carnegie Mellon University, CMU-CS-03-172, 2003.
27. L. Kolotilina and A. Yeremin, *Factorized sparse approximate inverse preconditionings* I. *Theory*, SIAM J. Matrix Anal. Appl., 14 (1993), pp. 45–58. MR1199543 (93m:65061)
28. S. Kuo and R. Mammone, *Image restoration by convex projections using adaptive constraints and the* 1 *norm*, IEEE Trans. Signal Process., 40 (1992), pp. 1159–1168.

29. Y. Li and F. Santosa, *A computational algorithm for minimizing total variation in image restoration*, IEEE Trans. Image Process., 6 (1996), pp. 987–995.
30. F.-R. Lin, M. K. Ng, and W.-K. Ching, *Factorized banded inverse preconditioners for matrices with Toeplitz structure*, SIAM J. Sci. Comput., 26 (2005), pp. 1852–1870. MR2196579 (2006i:65051)
31. C. van Loan, *Computational Frameworks for the Fast Fourier Transform*, SIAM, Philadelphia, 1992. MR1153025 (93a:65186)
32. R. Mammone and G. Eichmann, *Superresolving image restoration using linear programming*, Appl. Opt., 21 (1983), pp. 496–501.
33. N. Mastronardi, P. Lemmerling, A. Kalsi, D. O'Leary, and S. van Huffel, *Implemen- tation of regularized structured total least squares algorithms for blind image deblurring*, Linear Algebra Appl., 391 (2004), pp. 203–221. MR2094609 (2006h:94013)
34. V. Mesarovi'c, N. Galatsanos, and A. Katsaggelos, *Regularized constrained total least squares image restoration*, IEEE Trans. Image Process., 4 (1995), pp. 1096–1108.
35. T. Miyashita, *Super-resolved image restoration of holographic images by* 1*-norm minimization with clutter rejection*, Acoustical Imaging, 19 (1992), pp. 77–82.
36. M. Ng, R. Chan, and W. Tang, *A fast algorithm for deblurring models with Neumann boundary conditions*, SIAM J. Sci. Comput., 21 (1999), pp. 851–866. MR1718798 (2000g:65025)
37. M. K. Ng, R. J. Plemmons, and F. Pimentel, *A new approach to constrained total least squares image restoration*, Linear Algebra Appl., 316 (2000), pp. 237–258. MR1783308 (2001i:94012)
38. M. Nikolova, *Minimizers of cost-functions involving nonsmooth data-fidelity terms. Application to the processing of outliers*, SIAM J. Numer. Anal., 40 (2002), pp. 965–994. MR1949401 (2003k:62170)
39. M. Nikolova, *A variational approach to remove outliers and impulse noise*, J. Math. Imaging Vision, 20 (2004), pp. 99–120. MR2049784 (2005b:94006)
40. J. Nocedal and S. Wright, *Numerical Optimization*, Springer, New York, 1999. MR1713114 (2001b:90002)
41. C. Paige and M. Saunders, *Solution of sparse indefinite systems of linear equations*, SIAM J. Numer. Anal., 12 (1975), pp. 617–629. MR0383715 (52 #4595)
42. A. Pruessner and D. O'Leary, *Blind deconvolution using a regularized structured total least norm algorithm*, SIAM J. Matrix Anal. Appl., 24 (2003), pp. 1018–1037. MR2003319 (2004d:65053)
43. L. Rudin, S. Osher, and E. Fatemi, *Nonlinear total variation based noise removal algorithms*, Phys. D, 60 (1992), pp. 259–268.
44. S. Ruzinsky, *Sequential Least Absolute Deviation Estimation of Autoregressive Parameters*, Ph.D. thesis, Illinois Institute of Technology, Chicago, IL, 1989.
45. Y. Saad and M. Schultz, *GMRES: A generalized minimal residual algorithm for solving nonsymmetric linear systems*, SIAM J. Sci. Statist. Comput., 7 (1986), pp. 856–869. MR0848568 (87g:65064)
46. M. Saunders, *Interior Methods for Optimization with Application to Maximum Entropy Problems*, http://sccm.stanford.edu/csri/Saunders.pdf.
47. W. Tang, *Toward an effective sparse approximate inverse preconditioner*, SIAM J. Matrix Anal.

Appl., 20 (1998), pp. 970–986. MR1699789 (2000f:65045)
48. A. Tikhonov and V. Arsenin, *Solution of Ill-Posed Problems*, Winston, Washington, DC, 1977. MR0455365 (56 #13604)
49. S. Wright, *Primal-Dual Interior-Point Methods*, SIAM, Philadelphia, 1997. MR1422257 (98a:90004)

Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.

MR2196363 (2007a:35069) 35K55 (35K15 42B20 42C40 76-02 76D05 76F02)
Meyer, Yves (F-ENSET-AM)
Oscillating patterns in some nonlinear evolution equations.
Mathematical foundation of turbulent viscous flows, 101–187, *Lecture Notes in Math.*, 1871, *Springer*, *Berlin*, 2006.

This paper surveys in depth several of the claims and successes, but also limitations, surrounding the use of time-scale methods in solving nonlinear evolution equations. Importantly, in doing so, Meyer clarifies the role that wavelets can play in this field, and makes precise the parallels, but also subtle differences with Littlewood-Paley methods. When wavelets possess some modicum of regularity, they provide characterizations of function space norms in terms of magnitudes of wavelet coefficients, essentially whenever Littlewood-Paley dyadic decompositions characterize the given norm.

Meyer emphasizes here that, to understand well-posedness of evolution equations, one has to understand first the fundamental estimates required for existence and uniqueness. Then one can ask: is there a function space norm that characterizes when these estimates apply? If so, one can then ask whether that norm can be characterized, or nearly characterized, in terms of wavelets.

The survey begins with results pertaining to the "model" nonlinear heat equation

$$\frac{\partial u}{\partial t} = \Delta u + u^3$$

with

$$u(x,0) = u_0(x),$$

or a nonlinear Schrödinger equation (replacing $+u^3$ by $-u^3$), where $(x,t) \in \mathbb{R}^3 \times (0,\infty)$. This is followed by a detailed discussion of recent progress on the Navier-Stokes equations (NSE)

$$\frac{\partial u}{\partial t} = \Delta u - (v \cdot \nabla)v - \nabla p, \quad \nabla \cdot v = 0$$

with

$$v(x,0)=v_0(x),$$

where $v(x,t)\in\mathbb{R}^3$ denotes the velocity and p the pressure. Much of the progress discussed here is due to Meyer and several of his students, but Kato's foundational existence theory [see, e.g., T. Kato, Math. Z. **187** (1984), no. 4, 471–480; MR0760047 (86b:35171)], work of Y. Giga and T. Miyakawa [Comm. Partial Differential Equations **14** (1989), no. 5, 577–618; MR0993821 (90e:35130)] on vortex filaments, H. Koch and D. Tataru's global existence theorem for small initial values in $\nabla\cdot\mathrm{BMO}$ [Adv. Math. **157** (2001), no. 1, 22–35; MR1808843 (2001m:35257)], and M. E. Schonbek's optimal global decay estimate [J. Amer. Math. Soc. **4** (1991), no. 3, 423–449; MR1103459 (92j:35148)] are among related results all tied together by a collection of techniques for describing and bounding oscillatory terms. It is difficult to pull out a typical result in this wide ranging discussion, but methods for bounding the oscillations

$$\int_0^t S(t-\tau)\mathbb{P}\partial_j(v_jv)(\cdot,\tau)\,d\tau$$

provide the main focal point. Here S denotes the heat semigroup, while $\mathbb{P}$ denotes projection onto divergence-free fields. This part of the paper is not about wavelets per se, but an interesting discussion of why wavelets can have at most limited success in numerical analysis of NSE is included.

The last technical part of the survey addresses recent progress on improved Gagliardo-Nirenberg inequalities and the space of functions of bounded variation. Here, a fundamental wavelet estimate due to A. Cohen et al. [Rev. Mat. Iberoamericana **19** (2003), no. 1, 235–263; MR1993422 (2004f:42051)] is used to prove the inequality

$$\|f\|_2\le C(\|f\|_{\mathrm{BV}}\|f\|_*)^{1/2}.$$

Here, $\|\cdot\|_*$ is the norm in the homogeneous Besov space $\dot{B}^{-1,\infty}_\infty$ of distributions such that $|\langle f,g_{(a,b)}\rangle|$ are uniformly bounded, where $g=\exp(-\pi|x|^2)$ and $g_{(a,b,)}(x)=ag(a(x-b))$. Meyer also provides a "non-wavelet" proof of this estimate which, fortunately, was not discovered before refinements of the wavelet estimates, which are also outlined here.

{For the entire collection see MR2196359 (2006i:76001)}

Reviewed by *Joseph D. Lakey*

MR2196359 (2006i:76001) 76-06 (35-06 35Q30 76D03 76D05 76F02 76N10)

Constantin, P. (1-CHI); **Gallavotti, G.** (I-ROME-DP); **Kazhikhov, A. V.** (RS-AOSSI-HY); **Meyer, Y.** (F-ENSET-AM); **Ukai, S.** (J-YOKOT-AM)

★**Mathematical foundation of turbulent viscous flows.**

Lectures from the International C.I.M.E. (Centro Internazionale Matematico Estivo) Summer School held in Martina Franca, September 1–5, 2003.
Edited by M. Cannone and T. Miyakawa.
Lecture Notes in Mathematics, 1871.
Springer-Verlag, Berlin, 2006. *x*+253 *pp*. $59.95. *ISBN* 978-3-540-28586-1; 3-540-28586-5

Contents: Peter Constantin, Euler equations, Navier-Stokes equations and turbulence (1–43) MR2196360 (2007c:76001); Giovanni Gallavotti, CKN theory of singularities of weak solutions of the Navier-Stokes equations (45–74) MR2196361 (2006m:76031); Alexandre V. Kazhikhov [A. V. Kazhikhov], Approximation of weak limits and related problems (75–100) MR2196362 (2006m:35011); Yves Meyer, Oscillating patterns in some nonlinear evolution equations (101–187) MR2196363 (2007a:35069); Seiji Ukai, Asymptotic analysis of fluid equations (189–250) MR2196364 (2007m:76001).

{The papers are being reviewed individually.}

MR2206895 43-03 (42C40 94A12)

Meyer, Yves (F-ENSET-AM)

From wavelets to atoms. (English summary)

150 *years of mathematics at Washington University in St. Louis*, 105–117, *Contemp. Math.*, 395, *Amer. Math. Soc.*, *Providence*, *RI*, 2006.

{This item will not be reviewed individually. For details of the collection in which this item appears see MR2205840 (2006i:00013) .}

{For the entire collection see MR2205840 (2006i:00013)}

MR2172785 (2006k:90132) 90C31 (49J40 49J52 49K40)
Durand, S. [Durand, Sylvain] (F-PCRD-LFD);
Nikolova, M. [Nikolova, Mila] (F-ENSET-AM)

Stability of the minimizers of least squares with a non-convex regularization. I. Local behavior. (English summary)

Appl. Math. Optim. **53** (2006), *no. 2,* 185–208.

Many estimation problems amount to minimizing a piecewise $\mathcal{C}^m$ objective function, with $m \geq 2$, composed of a quadratic data-fidelity term and a general regularization term. It is widely accepted that the minimizers obtained using non-convex and possibly non-smooth regularization terms are frequently good estimates. However, few facts are known on the ways to control properties of these minimizers. The authors' work is dedicated to the stability of the minimizers of such objective functions with respect to variations of the data. In this paper they mainly consider the local case. Focusing on data points such that every local minimizer is isolated and results from a $\mathcal{C}^{m-1}$ local minimizer function (defined on some neighborhood), they show that all data points for which this fails form a set whose closure is negligible. For results in the global case see Part II of this study [S. Durand and M. Nikolova, Appl. Math. Optim. **53** (2006), no. 3, 259–277; MR2218728].

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References

1. F. Alizadeh, J.-P. Haeberly, and M. Overton, Complementarity and non-degeracy in semi-definite programming, Mathematical Programming, Series B, 77 (1997), 111–128. MR1461378 (98h:90072)
2. S. Alliney, Digital filters as absolute norm regularizers, IEEE Transactions on Medical Imaging, MI-12 (1993), 173–181.
3. L. Alvarez, P. L. Lions, and J. M. Morel, Image selective smoothing and edge detection by non-linear diffusion (II), SIAM Journal on Numerical Analysis, 29 (1992), 845–866. MR1163360 (93a:35072)
4. G. Aubert and P. Kornprobst, Mathematical Problems in Images Processing, Springer-Verlag, Berlin, 2002. MR1865346 (2002m:49001)
5. A. Avez, Calcul différentiel, Masson, Paris, 1991. MR0700398 (85c:58001)
6. J. E. Besag, On the statistical analysis of dirty pictures (with discussion), Journal of the Royal Statistical Society B, 48 (1986), 259–302. MR0876840 (88c:68079)
7. J. E. Besag, Digital image processing: towards Bayesian image analysis, Journal of Applied Statistics, 16 (1989), 395–407.
8. M. Black, G. Sapiro, D. Marimont, and D. Heeger, Robust anisotropic diffusion, IEEE Transactions on Image Processing, 7 (1998), 421–432.
9. A. Blake, Comparison of the efficiency of deterministic and stochastic algorithms for visual reconstruction, IEEE Transactions on Pattern Analysis and Machine Intelligence, PAMI-11 (1989), 2–12.
10. C. Bouman and K. Sauer, A generalized Gaussian image model for edge-preserving map estimation, IEEE Transactions on Image Processing, 2 (1993), 296–310.

11. V. Caselles, J.-M. Morel, and C. Sbert, An axiomatic approach to image interpolation, IEEE Transactions on Image Processing, 7 (1998), 59–83. MR1669524 (2000d:94001)
12. F. Catte, T. Coll, P. L. Lions, and J. M. Morel, Image selective smoothing and edge detection by nonlinear diffusion (I), SIAM Journal on Numerical Analysis, 29 (1992), 182–193. MR1149092 (93a:94005)
13. T.F. Chan and C.K. Wong, Total variation blind deconvolution, IEEE Transactions on Image Processing, 7 (1998), 370–375.
14. P. Charbonnier, L. Blanc-Féraud, and M. Barlaud, An adaptive reconstruction method involving discontinuities, in Proceedings of IEEE ICASSP, vol. V, 1993, 491–494.
15. P.G. Ciarlet, Introduction à l'analyse numérique matricielle et à l'optimisation, Collection mathématiques appliquées pour la maîtrise, Masson, Paris, 1990.
16. J.-N. Corvellec and A. Hantoute, Homotopical stability of isomated critical points of continuous functionals, Set-Valued Analysis, 10(2) (2002), 143–164. MR1926378 (2003f:49051)
17. G. Demoment, Image reconstruction and restoration: overview of common estimation structure and problems, IEEE Transactions on Acoustics Speech and Signal Processing, ASSP-37 (1989), 2024–2036.
18. D. Dobson and F. Santosa, Recovery of blocky images from noisy and blurred data, SIAM Journal on Applied Mathematics, 56 (1996), 1181–1199. MR1398414 (97c:68166)
19. D. Donoho, I. Johnstone, J. Hoch, and A. Stern, Maximum entropy and the nearly black object, Journal of the Royal Statistical Society B, 54 (1992), 41–81. MR1157714 (93d:62010)
20. A. L. Dontchev and T. Zollezi, Well-Posed Optimization Problems, Springer-Verlag, New York, 1993. MR1239439 (95a:49002)
21. A. Fiacco and G. McCormic, Nonlinear Programming, Classics in Applied Mathematics, SIAM, Philadelphia, PA, 1990. MR1058438 (91d:90089)
22. D. Geman and G. Reynolds, Constrained restoration and recovery of discontinuities, IEEE Transactions on Pattern Analysis and Machine Intelligence, PAMI-14 (1992), 367–383.
23. S. Geman and D. Geman, Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images, IEEE Transactions on Pattern Analysis and Machine Intelligence, PAMI-6 (1984), 721–741.
24. S. Geman and D. E. McClure, Statistical methods for tomographic image reconstruction, in Proceedings of the 46th Session of the ISI, Bulletin of the ISI, vol. 52, 1987, pp. 22–26. MR1027188
25. P. J. Green, Bayesian reconstructions from emission tomography data using a modified em algorithm, IEEE Transactions on Medical Imaging, MI-9 (1990), 84–93.
26. M. A. Hasan, Bifurcation analysis for perturbations of the first-order conditions of optimality, Journal of Mathematical Analysis and Applications, 188(3) (1994), 901–927. MR1305493 (95i:90086)
27. A. Ioffe and E. Schwartzman, Metric critical point theory 1, morse regularity and homotopic stability of a minimum, Journal de Mathématiques Pures et Appliquées, 75 (1996), 125–153. MR1380672 (97d:58046)
28. H. T. Jongen, P. Jonker, and T. F., Nonlinear Optimization in Finite Dimensions, 1st edn., Kluwer, Dordrecht, 1986.

29. D. Klatte and B. Kummer, Nonsmooth Equations in Optimization, Kluwer, Dordrecht, London, 2002. MR1909427 (2003c:49001)
30. Y. Leclerc, Constructing simple stable description for image partitioning, International Journal of Computer Vision, 3 (1989), 73–102.
31. J. Marroquin, S. Mitter, and T. Poggio, Probabilistic solution of ill-posed problems in computational vision, Journal of the American Statistical Association, 82 (1987), 76–89.
32. R. Mifflin, Semismooth and semiconvex functions in constrained optimization, SIAM Journal of Control and Optimization, 15 (1977), 959–972. MR0461556 (57 #1541)
33. J. Milnor, Topology from the Differential Point of View, The University Press of Virginia, Charlottesville, VA, 1965. MR0226651 (37 #2239)
34. D. Mumford and J. Shah, Boundary detection by minimizing functionals, in Proceedings of IEEEICASSP, 1985, pp. 22–26.
35. M. Nikolova, Markovian reconstruction using a GNC approach, IEEE Transactions on Image Processing, 8 (1999), 1204–1220.
36. M. Nikolova, Weakly constrained minimization. Application to the estimation of images and signals involving constant regions, Journal of Mathematical Imaging and Vision, 21 (2004), 155–175. MR2090130 (2005g:94020)
37. G. Pataki, On the rank of extreme matrices in semi-definite programs and the multiplicity of optimal eigenvalues, Mathematics of Operations Research, 23(2) (1998), 339–358. MR1626662 (99e:90094)
38. P. Perona and J. Malik, Scale-space and edge detection using anisotropic diffusion, IEEE Transactions on Pattern Analysis and Machine Intelligence, PAMI-12 (1990), 629–639.
39. G. d. Rham, Variétés différentiables, Hermann, Paris, 1955.
40. R. T. Rockafellar and J. B. Wets, Variational Analysis, Springer-Verlag, New York, 1997.
41. L. Rudin, S. Osher, and C. Fatemi, Nonlinear total variation based noise removal algorithm, Physica, 60 D (1992), 259–268.
42. A. Shapiro and M. K. H. Fan, On eigenvalue optimization, SIAM Journal on Optimization, 3 (1995), 552–568. MR1344670 (96m:90095)
43. M. A. Spivak, A Comprehensive Introduction to Differential Geometry, vol. 1, Brandeis University, Waltham, MA, 1970.
44. S. Teboul, L. Blanc-Féraud, G. Aubert, and M. Barlaud, Variational approach for edge-preserving regularization using coupled pde's, IEEE Transactions on Image Processing, 7 (1998), 387–397.
45. A. Tikhonov and V. Arsenin, Solutions of Ill-Posed Problems, Winston, Washington, DC, 1977. MR0455365 (56 #13604)
46. J. Weickert, Anisotropic Diffusion in Image Processing, B.G. Teubner, Stuttgart, 1998. MR1666943 (2000a:94003)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.